International Journal of Mathematical Analysis and Research
ISSN: 3108-2637 | Volume 2 Issue 2, 06-11, April-June 2026
DOI: https://doi.org/10.64137/3108-2637/IJMAR-V2I2P102

Received: 19/03/2026 Revised: 03/04/2026 Accepted: 08/04/2026 Published: 15/04/2026

Original Article

Development of a Reduced Fuzzy Soft Set Mathematical Framework for Transforming Fuzzified Acoustic Voice Data into Parameterized Diagnostic Structures

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ABSTRACT: *This paper analyses the development of a reduced fuzzy soft mathematical framework for modelling and detecting vocal disorder risk using clinical data. The paper presents a mathematically grounded fuzzy soft expert system for the quantitative assessment of vocal disorder risk based on acoustic and demographic parameters. Let* $U = \{x_1, x_2, x_3, \ldots, x_r\}$ *denote a finite universe of patients and* $E = \{F_0, VPI, Age\}$*, denote the parameter set comprising fundamental frequency, vocal perturbation index, and age. Clinical acoustic data were collected and represented as a dataset* $D = \{(x, F_0(x), VPI(x), Age(x)) | x \in U\}$. *Linguistic variables were obtained by the process of fuzzification using the appropriate designed membership functions assessment model. For each* $e \in E$*, a fuzzy membership function* $\mu_e : U \to [0,1]$ *is constructed using continuous triangular membership function to represent linguistic classifications. The fuzzified linguistic variables were further transformed into soft set using discrete α-cut levels. Discrete α-cut levels are employed to generate parameterized soft sets, which are subsequently reduced using redundancy elimination criteria derived from soft set theory. The reduced parameter sets preserve essential diagnostic information while minimizing computational complexity. The resulting mathematical framework provides a systematic method for organizing acoustic information without enforcing binary classification. The mathematical formulation preserves uncertainty, enables modelling of multiple parameter interaction and prepares the dataset for advanced diagnostic inference. The mathematically technique is consistent, scalable, and suitable for biomedical applications involving imprecise data.*

KEYWORDS: *Fuzzy- Soft Set Framework, Vocal Disorder Assessment, Fuzzification, Clinical Acoustic Analysis, A-Cut Analysis, Parameter Reduction.*

1. INTRODUCTION

Human voice production is a nonlinear biomechanical process governed by aerodynamic forces, neuromuscular coordination, and structural vibration of the vocal folds. Acoustic parameters such as fundamental frequency (F_0), vocal perturbation index (VPI) and Age are widely used in clinical vocal analysis due to their sensitivity to vocal fold behavior. However, their interpretation remains challenging because vocal characteristics do not exhibit rigid boundaries between normal and pathological states. Traditional diagnosis approaches often rely on crisp thresholds in vocal behaviors, which may fail to capture gradual transitions in vocal behavior, especially in early stage conditions. This limitation has motivated the application of uncertainty modelling techniques in vocal health analysis.

One of the most influential mathematical tools for handling uncertainty is fuzzy set theory, introduced by Zadeh, (1965), which allows elements to belong to sets with varying degrees of membership. This approach has been extensively applied in biomedical systems to represent gradual transitions and imprecise measurements (Ross, 2010; Zimmermann, 2010). In vocal analysis, fuzzy modelling has been shown to provide a more realistic interpretation of acoustic variability compare to rigid classification schemes (Titze, 1994; Baken & Orlikoff, 2000). In acoustic voice analysis, the transformation of continuous signals into interpretable mathematical structures remains a significant challenge. While fuzzification provides a mechanism for representing uncertainty through graded membership values, it does not fully address the need for parameterized organization of such information. This limitation necessitates the integration of fuzzified representations with a structure capable of handling multiple parameter relationships.

Fuzzy set theory (Zadeh, 1965) introduced graded membership to model such gradual transitions. Through fuzzification, crisp acoustic measurements are transformed into graded linguistic variables, enabling a more realistic representation of vocal behavior compared to rigid threshold-based classification.To further enhanced parameter based modelling, Molodtsov (1999) introduced soft set theory as a parameterized approach to uncertainty modelling. Unlike fuzzy sets, soft sets do not require membership tuning and has been successfully applied in decision making and information systems. Maji et al. (2003) introduced fuzzy soft sets by integrating fuzzy membership into soft parameterization. The model has been applied in areas like medical diagnosis, pattern recognition, and decision support systems. In the context of vocal health, recent studies have demonstrated the effectiveness of fuzzy soft techniques in modelling uncertainty and supporting diagnosis reasoning (Sanabria et al., 2023). Moreover, parameter reduction approaches have been explored to improve the efficiency of fuzzy soft systems. Methods based on correlation measures (Ma et al., 2020) and distance metrics (Qin et al., 2023) have been proposed to eliminate redundant parameters while preserving essential information. These techniques highlight the importance of constructing minimal yet informative parameter sets. Despite these advancements, a critical gap remains in the modelling pipeline. Most existing works by Okigbo et al. (2026) focus on transforming acoustic data into fuzzy linguistic representations and subsequently applying decision making or assessment models. However, the intermediate stage of transforming fuzzified data into a structured, parameterized form suitable for soft set analysis is often overlooked. This study addresses this gap by transforming fuzzified outputs into a soft set representation using α-cut and parameter reduction. This work serves as a continuation of prior fuzzification research, focusing strictly on structural transformation.

2. PRELIMINARIES

This section briefly presents the basic mathematical concepts underlying the proposed transformation techniques. Emphasis is placed on the transformation mechanism, while standard definitions are presented in a concise and referenced form.

2.1. FUZZY REPRESENTATION OF DATA

In vocal analysis, acoustic measurements do not belong strictly to a single category. Instead, they exhibit gradual transitions. Fuzzy set theory provides a natural way to model this behavior by allowing partial membership. A fuzzy mapping is defined mathematically as:

$$\mu: U \rightarrow [0,1] \tag{2.1}$$

Where $\mu(x)$ represents the degree to which an element $x \in U$ belongs to a given category (Zadeh, 1965)

2.2. SOFT SET REPRESENTATION

While fuzzy sets describe degrees of belonging, they do not provide structured grouping. Soft set theory introduces a parameter-based organization of elements, enabling structured representation. A soft set is defined mathematically as:

Let $P(U)$ denotes the power set of$U$, i.e., the set of all subsets of U.

$$F: E \rightarrow P(U) \tag{2.2}$$

Where each parameter $e \in E$ is associated with a subset of the universe (Molodtsov, 1999)

2.3. A-CUT TRANSFORMATION

A-Cut operators are used to bridge fuzzy and soft representations. These operators extract elements that satisfy a specified confidence level, thereby converting fuzzy values into crisp subsets (Zadeh, 1965). Mathematically, α-Cut is defined to be:

$$[A_\alpha] = \{x \in U | \mu(x) \geq \alpha\}, \quad \alpha \in [0,1] \tag{2.3}$$

2.4. FUZZY-SOFT SET THEORY

Fuzzy soft set theory was introduced by Maji et al. (2003) and Ali et al. (2009) as a hybrid approach that combines fuzzy set theory and soft set theory to handle both vagueness and parameter uncertainty in decision-making. This step connects fuzzy representation to structured grouping by applying α thresholds to each parameter. Mathematically, fuzzy soft set is defined to be:

Given a universal set U and a parameter set E,

$$F = \{(e, \mu_{F(e)}(x)) \mid e \in E, x \in U\} \tag{2.4}$$

Where $\mu_{F(e)}(x) : U \rightarrow [0,1]$ is a fuzzy membership function for each parameter e.

Example 2.1: Let $x_i = (F_{0_i}, VPI_i, AGE_i)$

Suppose$\mu_{F_0}(x_1) = 0.5, \quad \mu_{VPI}(x_1) = 0.9$

Then

$(x_1, 0.5) \in F(F_O), \quad (x_1, 0.9) \in F(VPI)$

2.5. PARAMETER REDUCTION

Reductant α-levels that produce identical subsets are removed to simplify the model.

Mathematical formulation is given as:

$$F_e(\alpha_i) \subseteq F_e(\alpha_j). \tag{2.5}$$

$$E^* = \{(e, \alpha) | non - redundant\}$$

3. MATERIALS AND METHODS

In this section, the focus shifts from describing uncertainty to organizing it. While the fuzzification stage (Okigbo et al., 2026) provided a detailed representation of vocal features using membership values, those results remain inherently unstructured. Each patient belongs to several categories to varying degrees, but there is no direct way to group or compare patients in a systematic manner.

The goal of this stage is to transform fuzzified vocal data into structured subsets that can be easily analyzed. The transformation procedures are computed as follows: select an α level, identify qualifying patients, form subsets, compare subsets, and remove reductancy. Instead of working directly with membership values, we organize patients into groups based on confidence levels.

3.1. SOURCE OF DATA

The dataset used in this study is derived from the previously established fuzzification process reported by Okigbo et al. (2026). In that work, each patient was represented using membership values corresponding to linguistic categories associated with acoustic parameters and age. This study adopts those results directly as input.

3.2. MATHEMATICAL REPRESENTATION

Let

U= $\{x_1, x_2, x_3, \ldots, x_{20}\}$

Denote the set of patients under investigation. Each patient, x_i with fuzzy membership value cross is represented as:

$x_i = (F_{0_i}(AT, NT, ST), VPI_i(NV, RV, AV), AGE_i(Y, M, O))$

3.3. α THRESHOLD SELECTION

Multiple thresholds are selected and used to observe how grouping changes under varying levels of strictness. Discretization improves interpretability and efficiency (Pedrycz & Gomide, 2013). Based on α-cut theory, the threshold sets for each acoustic vocal parameter are given below:

F_0 = {0.04, 0.20, 0.36, 0.52, 0.68}

VPI = {0.02, 0.30, 0.58, 0.86, 0.98}

Age = {0, 0.25, 0.50, 0.75, 1}

These values partition [0,1] into interpretable diagnostic levels.

3.4. RESULTS

Following the fuzzification results of 20 patients obtained in the preceding study by Okigbo et al. (2026), where each patient was represented using graded membership values across acoustic and demographic parameters, the next step involves transforming these fuzzy representations into structured subsets. The fuzzified dataset (see Table 2 in Okigbo et al., 2026) serve as a foundation for this transformation. Using these membership values, α-cut thresholds are applied to extract patient groupings at different confidence levels, thereby enabling the construction of a structured soft set representation.

3.5. ILLUSTRATIVE TRANSFORMATION USING SELECTED PATIENTS

For clarity and illustration of the transformation procedure, a subset of five patients is selected from the fuzzified dataset reported by Okigbo et al. (2026). While the complete dataset is used for the overall analysis, this illustrative example demonstrates how fuzzy membership values are systematically converted into structured soft set representations using α-cut thresholds. The selected fuzzified membership values are shown in table 3.1

TABLE 1 Extracted Fuzzified Membership Values (Okigbo et al., 2026)

Patient	AT	NT	ST	NV	RV	AV	Y	M	O
x_1	0.0	0.0	0.341	0.659	0.0	0.0	0.0	0.0	0.8
x_2	0.0	0.400	0.0	0.0	0.790	0.0	0.6	0.0	0.0
x_3	0.0	0.729	0.0	0.677	0.0	0.0	0.0	0.8	0.0
x_4	0.0	0.280	0.0	0.830	0.0	0.0	0.0	0.8	0.0
x_5	0.126	0.0	0.0	0.571	0.0	0.0	0.0	0.0	0.8

To obtain a unified representation for each acoustic parameter, the soft set transformation results corresponding to individual linguistic variables were aggregated at each α-cut threshold. For instance, in the case of fundamental frequency (F_0), the subsets obtained for Abnormal Tone(AT), Normal Tone (NT), and Stable Tone (ST) were combined using set union to form a single parameter level subset. This is expressed as:

$F_{0_\alpha} = \{AT_\alpha \cup NT_\alpha \cup ST_\alpha\}$.

A similar aggregation was performed for the vocal perturbation index (VPI) and Age parameters.

$VPI_\alpha = \{NV_\alpha \cup RV_\alpha \cup AV_\alpha\}$.

$Age_\alpha = \{Y_\alpha \cup M_\alpha \cup O_\alpha\}$.

This aggregation provides a consolidated view of patient inclusion at each α level, allowing for clearer comparison across parameters. Hence, some of the soft sets obtained by choosing parameter sets using the membership functions are summarized in the tabular form below:

TABLE 2 Soft Set Transformation Results Before Parameter Reduction.

Parameter	α-cut Threshold	Soft Set Result
F_0	0.04	$\{x_1, x_2, x_3, x_4, x_5\}$
F_0	0.20	$\{x_1, x_2, x_3, x_4\}$
F_0	0.36	$\{x_2, x_3\}$
F_0	0.52	$\{x_3\}$
F_0	0.68	$\{x_3\}$
VPI	0.02	$\{x_1, x_2, x_3, x_4, x_5\}$
VPI	0.30	$\{x_1, x_2, x_3, x_4, x_5\}$
VPI	0.58	$\{x_1, x_2, x_3, x_4\}$
VPI	0.86	$\emptyset$
VPI	0.98	$\emptyset$
Age	0	$\{x_1, x_2, x_3, x_4, x_5\}$
Age	0.25	$\{x_1, x_2, x_3, x_4, x_5\}$
Age	0.50	$\{x_1, x_2, x_3, x_4, x_5\}$
Age	0.75	$\{x_1, x_3, x_4, x_5\}$
Age	1	$\emptyset$

To improve clarity, the above table can be interpreted below;

α-level soft set for F_0

Let $\mu_{F_0}(x_i)$ be the fuzzified membership value of F_O for patient x_i.

$F_{0_{0.04}} = \{x_i \in U : \mu_{F_0}(x_i) \geq 0.04, 0.04 \in (0,1]\}$
$= \{x_1, x_2, x_3, x_4, x_5\}$

$F_{0_{0.20}} = \{x_i \in U : \mu_{F_0}(x_i) \geq 0.20, 0.20 \in (0,1]\}$
$= \{x_1, x_2, x_3, x_4\}$

$F_{0_{0.36}} = \{x_i \in U : \mu_{F_0}(x_i) \geq 0.36, 0.36 \in (0,1]\}$
$= \{x_2, x_3\}$

$F_{0_{0.52}} = \{x_i \in U : \mu_{F_0}(x_i) \geq 0.52, 0.52 \in (0,1]\}$
$= \{x_3\}$

$F_{0_{0.68}} = \{x_i \in U : \mu_{F_0}(x_i) \geq 0.68, 0.68 \in (0,1]\}$
$= \{x_3\}$

α-level soft set for VPI

Let $\mu_{VPI}(x_i)$ be the fuzzified membership value of VPI for patient x_i.

$F_{VPI_{0.02}} = \{x_i \in U : \mu_{VPI}(x_i) \geq 0.02, 0.02 \in (0,1]\}$
$= \{x_1, x_2, x_3, x_4, x_5\}$

$F_{VPI_{0.3}} = \{x_i \in U : \mu_{VPI}(x_i) \geq 0.3, 0.3 \in (0,1]\}$
$= \{x_1, x_2, x_3, x_4, x_5\}$

$F_{VPI_{0.58}} = \{x_i \in U : \mu_{VPI}(x_i) \geq 0.58, 0.58 \in (0,1]\}$
$= \{x_1, x_2, x_3, x_4\}$

$F_{VPI_{0.86}} = \{x_i \in U : \mu_{VPI}(x_i) \geq 0.86, 0.86 \in (0,1]\}$
$= \emptyset$

$F_{VPI_{0.98}} = \{x_i \in U : \mu_{VPI}(x_i) \geq 0.98, 0.98 \in (0,1]\}$
$= \emptyset$

α-level soft set for Age

Let $\mu_{AGE}(x_i)$ be the fuzzified membership value of AGE for patient x_i.

$F_{AGE_0} = \{x_i \in U : \mu_{AGE}(x_i) \geq 0, 0 \in (0,1]\}$
$= \{x_1, x_2, x_3, x_4, x_5\}$

$F_{AGE_{0.25}} = \{x_i \in U : \mu_{AGE}(x_i) \geq 0.25, 0.25 \in (0,1]\}$
$= \{x_1, x_2, x_3, x_4, x_5\}$

$F_{AGE_{0.5}} = \{x_i \in U : \mu_{AGE}(x_i) \geq 0.5, 0.5 \in (0,1]\}$
$= \{x_1, x_2, x_3, x_4, x_5\}$

$F_{AGE_{0.75}} = \{x_i \in U : \mu_{AGE}(x_i) \geq 0.75, 0.75 \in (0,1]\}$
$= \{x_1, x_3, x_4, x_5\}$

$F_{AGE_1} = \{x_i \in U : \mu_{AGE}(x_i) \geq 1, 1 \in (0,1]\}$
$= \emptyset$

3.6. PARAMETER REDUCTION RESULT

Following the construction of the soft set representation, a parameter reduction procedure was applied to eliminate redundancy without loss of information. Two parameters were considered redundant if they produced identical subsets under the same α-cut. The proposed method was applied to the complete dataset of twenty patients derived from the fuzzification stage. The resulting soft set representations for the full dataset obtained after eliminating redundant subsets are summarized in tabular form below:

TABLE 3 Reduced Soft Set Transformation Results After Parameter Reduction.

Parameter	α-cut Threshold	Reduced Soft Set Result
F_0	0.20	$\{x_1, x_2, x_3, x_4, x_6, x_7, x_8, x_9, x_{10}, x_{12}, x_{13}, x_{14}, x_{15}, x_{16}, x_{17}, x_{18}, x_{19}, x_{20}\}$
F_0	0.36	$\{x_2, x_3, x_6, x_8, x_{10}, x_{12}, x_{13}, x_{14}, x_{15}, x_{16}, x_{17}, x_{18}, x_{19}, x_{20}\}$
F_0	0.52	$\{x_3, x_6, x_8, x_{10}, x_{12}, x_{13}, x_{17}, x_{18}, x_{19}\}$
VPI	0.30	$\{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{12}, x_{13}, x_{11}, x_{14}, x_{15}, x_{16}, x_{17}, x_{19}\}$
VPI	0.58	$\{x_1, x_2, x_3, x_4, x_8, x_9, x_{13}, x_{14}, x_{16}, x_{17}, x_{19}\}$
VPI	0.86	$\{x_8, x_9, x_{16}, x_{13}, x_{17}, x_{19}\}$
Age	0.25	$\{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}, x_{11}, x_{12}, x_{13}, x_{14}, x_{15}, x_{16}, x_{17}, x_{19}, x_{20}\}$
Age	0.50	$\{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_{10}, x_{11}, x_{12}, x_{13}, x_{14}, x_{15}, x_{16}, x_{17}, x_{20}\}$
Age	0.75	$\{x_1, x_3, x_4, x_5, x_6, x_8, x_{10}, x_{16}, x_{17}, x_{20}\}$

3.7. MATHEMATICAL REPRESENTATION OF REDUCED SOFT SET RESULTS.

Following parameter reduction, the final soft set representations retained for the analysis are expressed as:

$F_{0_{0.52}} = \{x_3, x_6, x_8, x_{10}, x_{12}, x_{13}, x_{17}, x_{18}, x_{19}\}$

$F_{VPI_{0.86}} = \{x_8, x_9, x_{16}, x_{13}, x_{17}, x_{19}\}$

$F_{AGE_{0.75}} = \{x_1, x_3, x_4, x_5, x_6, x_8, x_{10}, x_{16}, x_{17}, x_{20}\}$

These selected subsets illustrate the progressive refinement achieved through parameter reduction. Higher α levels retain fewer patients, thereby isolating more strongly defined cases within each parameter group.

3.8. COMBINED GRAPHICAL ANALYSIS OF SOFT SET TRANSFORMATION RESULTS.

To enhance interpretation, the transformation results are illustrated graphically. The plot shows how patient groupings change across different α levels, providing a visual insight into the refinement of subsets as the confidence threshold increases.

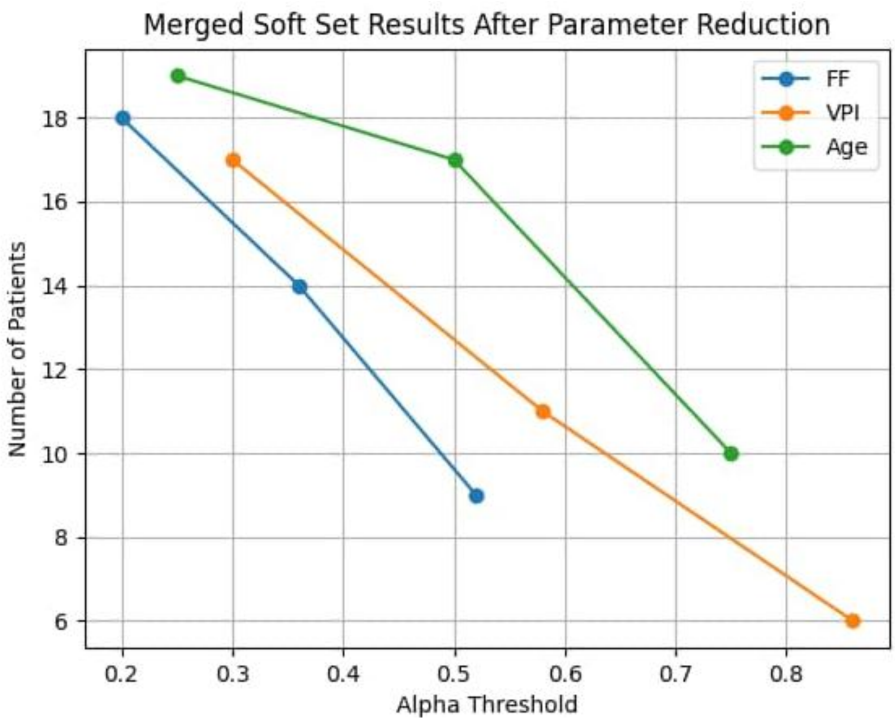

Figure 1:

As shown in Figure1, the number of patients decreases across all parameters as α increases, indicating progressive refinement of the subsets. Acoustic parameters show gradual changes, while age parameters exhibit sharper transitions, reflecting their categorical nature.

4. DISCUSSION

The results show a consistent pattern in how patient groupings change as the α threshold increases. Across all parameters, the number of patients decreases gradually, confirming that the transformation behaves in a structured and predictable way. The acoustic parameters; fundamental frequency (F_0) and vocal perturbation index (VPI) exhibit smooth transitions, indicating that vocal characteristics vary continuously rather than abruptly. In contrast, the age parameter shows sharper changes, reflecting its more categorical nature. This difference highlights the flexibility of the model in handling both gradual and discrete types of data. The α-cut transformation proves effective in converting fuzzy membership values into meaningful and interpretable subsets. By allowing analysis at different confidence levels, the model provides both broad and refined views of the data. In addition, parameter reduction simplifies the framework by removing reductant information while preserving the essential structure. The makes the model more efficient and easier to interpret. Therefore, the technique offers a practical way to organize uncertain vocal data into structured patterns that can support further analysis and decision making.

5. CONCLUSION

This study developed a reduced fuzzy soft set framework for transforming fuzzified vocal data into structured subsets using α-cut parameterization. The approach organizes uncertain membership values into clear and interpretable groupings while preserving their underlying variability. The results show consistent and meaningful patterns across parameters, and the inclusion of parameter reduction improves clarity by removing redundancy. Hence, the framework provides a practical and efficient basis for further analysis and potential diagnostic applications.

REFERENCES

[1] M. I. Ali, F. Feng, X. Liu, W. K. Min, and M. Shabir, “On some new operations in soft set theory,” *Computers & Mathematics with Applications*, vol. 57, no. 9, pp. 1547–1553, May 2009, doi: https://doi.org/10.1016/j.camwa.2008.11.009.

[2] “R. Baken and F. Orlikoff, ‘Clinical Measurement of Speech and Voice,’ 2nd Edition, Singular Publishing, San Diego, 2000. - References - Scientific Research Publishing,” *Scirp.org*, 2024. https://www.scirp.org/reference/referencespapers?referenceid=1028006

[3] X. Ma, K. Qin, and Y. Zhang, “Parameter reduction in fuzzy soft sets based on correlation measures,” *Applied Soft Computing*, vol. 89, 2020.

[4] “Maji, P.K., Roy, A.R. and Biswas, R. (2001) Fuzzy Soft Sets. Journal of Fuzzy Mathematics, 9, 589-602. - References - Scientific Research Publishing,” *Scirp.org*, 2026. https://www.scirp.org/reference/referencespapers?referenceid=1185371 (accessed May 23, 2026).

[5] “Molodtsov, D. (1999) Soft Set Theory—First Results. Computers & Mathematics with Applications, 37, 19-31. - References - Scientific Research Publishing,” *Scirp.org*, 2026. https://www.scirp.org/reference/referencespapers?referenceid=1520004 (accessed May 23, 2026).

[6] O. C. J, O. I. A., and A. A. O., “A Hybrid Fuzzy Soft Prediction Model for Transforming Acoustic Voice Data into Linguistic Knowledge,” *International Journal of Science and Research Archive*, vol. 18, no. 2, pp. 1035–1047, Feb. 2026, doi: https://doi.org/10.30574/ijsra.2026.18.2.0398.

[7] Witold Pedrycz and F. Gomide, Fuzzy Systems Engineering. John Wiley & Sons, 2013.

[8] H. Qin, Y. Wang, X. Ma, J. Wang, and C. Jiang, “A Euclidean Distance-based parameter reduction algorithm for interval-valued fuzzy soft sets,” *Expert Systems with Applications*, vol. 234, p. 121106, Dec. 2023, doi: https://doi.org/10.1016/j.eswa.2023.121106.

[9] “Ross, T.J. (2010) Fuzzy Logic with Engineering Applications. 3rd Edition, Wiley, Hoboken. - References - Scientific Research Publishing,” *Scirp.org*, 2026. https://www.scirp.org/reference/referencespapers?referenceid=1247688 (accessed May 23, 2026).

[10] J. Sanabria, Marinela Álvarez, and O. Ferrer, “Fuzzy Set and Soft Set Theories as Tools for Vocal Risk Diagnosis,” *Applied Computational Intelligence and Soft Computing*, vol. 2023, pp. 1–12, Nov. 2023, doi: https://doi.org/10.1155/2023/5525978.

[11] “I. R. Titze, ‘Principles of Voice Production,’ Prentice Hall, Boston, 1994. - References - Scientific Research Publishing,” *Scirp.org*, 2026. https://www.scirp.org/reference/referencespapers?referenceid=606514

[12] L. A. Zadeh, “Fuzzy sets,” *Information and Control*, vol. 8, no. 3, pp. 338–353, Jun. 1965, doi: https://doi.org/10.1016/s0019-9958(65)90241-x.

[13] H.-J. Zimmermann, *Fuzzy Set Theory—and Its Applications*. Springer Science & Business Media, 2001.