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Original Article

Genetic Algorithm Optimized Fuzzy N-Policy Queue Using α-Cut based Performance Analysis

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ABSTRACT: *In real-life queueing systems, many parameters such as arrival rate and service rate are often uncertain or imprecise. To handle such uncertainty, fuzzy set theory provides an effective framework for modelling queueing systems. In this paper, an N-policy M/M/1 queueing model is studied in a fuzzy environment where system parameters are represented as fuzzy numbers. The α-cut approach is employed to transform the fuzzy parameters into interval values, allowing the evaluation of system performance measures such as the expected queue length and waiting time. To obtain the optimal operating policy, a Genetic Algorithm (GA) is applied to determine the optimal threshold value of Nthat minimizes the total system cost. The proposed method combines fuzzy modeling with evolutionary optimization to improve decision-making under uncertainty. Numerical illustrations and graphical analysis are provided to demonstrate the effectiveness of the proposed approach. The results show that the integration of α-cut based fuzzy analysis with Genetic Algorithm optimization provides a flexible and efficient method for determining optimal policies in uncertain queueing environments.*

KEYWORDS: *Fuzzy Queueing System, N-Policy, Α-Cut Method, Genetic Algorithm, M/M/1 Queue, Optimization, Performance Measures.*

1. INTRODUCTION

Queueing theory plays an important role in analysing congestion and service systems such as computer networks, telecommunication systems, manufacturing systems, and transportation networks. Classical queueing models generally assume that system parameters such as arrival rate and service rate are precisely known. However, in many real-life situations, these parameters are uncertain or imprecise due to incomplete information or fluctuating environments. To address such uncertainty, fuzzy set theory provides a useful mathematical framework for representing imprecise information. In fuzzy queueing models, parameters like arrival rate and service rate can be represented by fuzzy numbers instead of fixed values. One common method used to analyse fuzzy systems is the α-cut technique, which converts fuzzy numbers into interval values and allows the evaluation of system performance measures. Chen (2005, 2006) [12] developed nonlinear programming approaches to analyse fuzzy queueing models using the α-cut method. This study developed a nonlinear programming approach to analyse fuzzy queueing systems where arrival and service rates are fuzzy numbers. The α-cut approach is used to transform the fuzzy queue into a set of crisp queueing models for evaluating performance measures. Among various service control policies, the N-policy queueing system is widely used to reduce operating costs. In this policy, the server remains idle until the number of customers in the system reaches a predetermined threshold value N. H. R. Sama, V. K. Vemuri, B. V. S. N. H. Prasad (2021). This paper studies an N-policy queue in a fuzzy environment and develops membership functions for cost and performance measures. The model shows how fuzzy parameters influence optimal control policies in queueing systems. Y. Saritha, K. Satish Kumar, K. Chandan, G. Sridhar, (2018) [11] their study analyses a fuzzy N-policy vacation queueing system where triangular and trapezoidal fuzzy numbers are used. The α-cut technique converts fuzzy parameters into crisp intervals. J. Alonge, R. M. Matendo, D. L. Okenge, (2023) This works analysed fuzzy Markovian queueing models using flexible α-cut techniques to compute performance measures. For N-policy, once the threshold is reached, the server begins service and continues until the system becomes empty again. Determining the optimal value of $N$is an important optimization problem. In this study, a Genetic Algorithm (GA) is employed to determine the optimal threshold value that minimizes the total system cost. Genetic Algorithms are powerful evolutionary optimization techniques inspired by natural selection and are widely used to solve complex optimization problems. K. S. Al-Sultan, Z. Hussain, (2018). The paper applies Genetic Algorithms for optimizing service systems and scheduling problems, showing how evolutionary algorithms can efficiently find optimal solutions in complex systems. This paper proposes a Genetic Algorithm optimized fuzzy N-policy M/M/1 queueing model using the α-cut approach. The fuzzy parameters are transformed into interval values using α-cuts, and system performance measures are evaluated. Numerical examples and graphical results are presented to demonstrate the effectiveness of the proposed method.

2. FUZZY QUEUEING MODEL DESCRIPTION

2.1. QUEUE MODEL

Consider an M/M/1 queueing system with N-policy customers arrive according to a Poisson μ. The system follows First-Come-First-Served (FCFS**)** discipline. The server remains idle until the number of customers reaches **N**. When the number of customers becomes N, the server starts service. Service continues until the system becomes empty, after which the server again waits for N customers.

The parameters λ and μ are considered fuzzy numbers to represent uncertainty. Because the parameters are fuzzy, the system performance measures such as average queue length and waiting time are also fuzzy in nature.

2.1.1 STATE DESCRIPTION OF THE SYSTEM

Let

$$n = \text{number of customers in the system}$$

The system can be described by the following states:

Idle State

When

$$0 \le n < N$$

the server remains **idle**, and customers continue to arrive.

Busy State

When

$$n \ge N$$

the server begins service and continues serving customers until the system becomes empty.

2.2. FUZZY INVOLVEMENT

In practical queueing systems, parameters such as arrival rate and service rate are often uncertain due to fluctuations in demand, environmental conditions, and incomplete information. To model such uncertainty, fuzzy set theory provides an effective mathematical framework. single-server M/M/1 queueing system with N-policy operating in a fuzzy environment. In this model, the arrival rate and service rate are represented by triangular fuzzy numbers. Among different types of fuzzy numbers, triangular fuzzy numbers are widely used because of their simplicity and computational efficiency. They require only three parameters representing the minimum, most likely, and maximum possible values of a variable. In addition, triangular fuzzy numbers are convenient for analytical calculations and α-cut based analysis. Therefore, in this study the arrival rate and service rate are represented as triangular fuzzy numbers to capture the uncertainty associated with the system parameters.

Let the fuzzy arrival rate be represented as

$$\tilde{\lambda} = (\lambda_1, \lambda_2, \lambda_3)$$

where

- λ_1 denotes the minimum possible arrival rate
- λ_2 denotes the most probable arrival rate
- λ_3 denotes the maximum possible arrival rate

Let the fuzzy service rate be represented as

$$\tilde{\mu} = (\mu_1, \mu_2, \mu_3)$$

where

- μ_1 represents the minimum service rate
- μ_2 represents the most likely service rate
- μ_3 represents the maximum service rate

2.2.1. MEMBERSHIP FUNCTION AS

The membership function of the triangular fuzzy arrival rate $\tilde{\lambda}$ is given by

$$\mu_{\tilde{\lambda}}(x) = \begin{cases} 0, & x < \lambda_1 \\ \dfrac{x-\lambda_1}{\lambda_2-\lambda_1}, & \lambda_1 \le x \le \lambda_2 \\ \dfrac{\lambda_3-x}{\lambda_3-\lambda_2}, & \lambda_2 \le x \le \lambda_3 \\ 0, & x > \lambda_3 \end{cases}$$

Similarly, the membership function of the triangular fuzzy service rate $\tilde{\mu}$ is defined as

$$\mu_{\tilde{\mu}}(x) = \begin{cases} 0, & x < \mu_1 \\ \dfrac{x-\mu_1}{\mu_2-\mu_1}, & \mu_1 \le x \le \mu_2 \\ \dfrac{\mu_3-x}{\mu_3-\mu_2}, & \mu_2 \le x \le \mu_3 \\ 0, & x > \mu_3 \end{cases}$$

2.2.2. ALPHA-CUT REPRESENTATION

Using the α-cut technique, the triangular fuzzy arrival and service rates are converted into interval values.

For a triangular fuzzy number (a_1, a_2, a_3), the α-cut interval is defined as

$$A_\alpha = [a_1 + \alpha(a_2 - a_1),\ a_3 - \alpha(a_3 - a_2)]$$

where

$$0 \leq \alpha \leq 1$$

Then or the fuzzy arrival rate

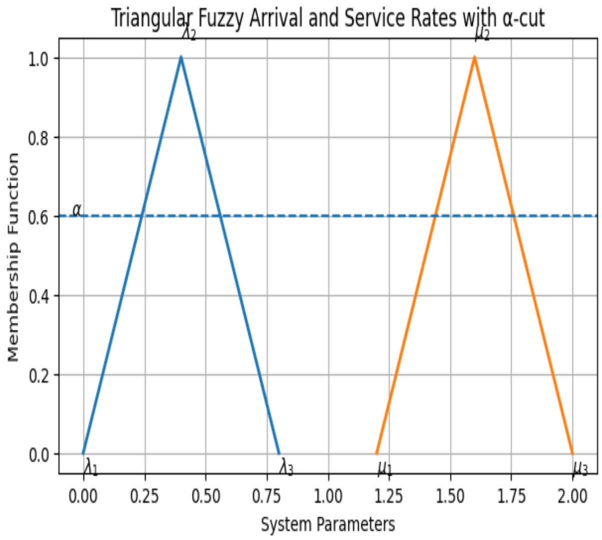
$$\lambda^\alpha = [\lambda_1 + \alpha(\lambda_2 - \lambda_1),\ \lambda_3 - \alpha(\lambda_3 - \lambda_2)]$$

For the fuzzy service rate

$$\mu^\alpha = [\mu_1 + \alpha(\mu_2 - \mu_1),\ \mu_3 - \alpha(\mu_3 - \mu_2)]$$

Where

$$0 \leq \alpha \leq 1$$



3. PERFORMANCE MEASURES

The following performance measures are considered:
Expected number of customers in the system

$$L_s$$

Expected number of customers in the queue

$$L_q$$

Average waiting time in the system

$$W_s = \frac{L_s}{\lambda}$$

Average waiting time in the queue

$$W_q = \frac{L_q}{\lambda}$$

These measures are evaluated using the interval values obtained from the α-cut representation of fuzzy parameters.

Cost Function

The total expected cost of the system is defined as

$$TC(N) = C_h L_s + C_s$$

where

- C_h= holding cost per customer
- C_s= service cost per unit time
- L_s= expected number of customers in the system

The objective is to determine the optimal threshold value N that minimizes the total cost.

4. OPTIMIZATION USING GENETIC ALGORITHM

A Genetic Algorithm is used to determine the optimal threshold value N. The algorithm generates a population of candidate solutions and evaluates them using the cost function. Through evolutionary operations such as selection, crossover, and mutation, the algorithm iteratively improves the solutions until the minimum cost is obtained.

The GA procedure consists of the following steps:

1. ***Initialization***
   Generate an initial population of possible values of N.
2. ***Fitness***
   Evaluate each solution using the total cost function.
3. $Fitness = \frac{1}{TC(N)}$
4. ***Selection***
   Select the best individuals for reproduction.
5. ***Crossover***
   Combine parent solutions to create new offspring.
6. ***Mutation***
   Introduce small random changes to maintain diversity.
7. ***Termination***
   The algorithm stops when the optimal or near-optimal value of $N$is obtained.
8. The optimal threshold obtained through GA provides the **minimum system cost under fuzzy conditions**. In the proposed model, the Genetic Algorithm is employed to determine the optimal threshold value of Nthat minimizes the total operating cost of the system. Each chromosome represents a candidate value of N, and the fitness of each chromosome is evaluated using the total cost function derived from the queue performance measures. Through iterative processes of selection, crossover, and mutation, the algorithm converges to the optimal solution N^*.

5. NUMERICAL ILLUSTRATION

Assume that the arrival and service rates of the queueing system are represented by triangular fuzzy numbers.

$$\tilde{\lambda} = (2,4,6)$$
$$\tilde{\mu} = (7,9,11)$$

where

- $\lambda_1 = 2, \lambda_2 = 4, \lambda_3 = 6$
- $\mu_1 = 7, \mu_2 = 9, \mu_3 = 1$

For a triangular fuzzy number:

$$\lambda^\alpha = [\lambda_L(\alpha), \lambda_U(\alpha)]$$

where

$$\lambda_L(\alpha) = \lambda_1 + \alpha(\lambda_2 - \lambda_1)$$
$$\lambda_U(\alpha) = \lambda_3 - \alpha(\lambda_3 - \lambda_2)$$

Similarly for service rate:

$$\mu_L(\alpha) = \mu_1 + \alpha(\mu_2 - \mu_1)$$
$$\mu_U(\alpha) = \mu_3 - \alpha(\mu_3 - \mu_2)$$

This method comes from the α-cut technique used in Fuzzy Logic.

α-cut Table

α	$\lambda L(\alpha)$	$\lambda U(\alpha)$	$\mu L(\alpha)$	$\mu U(\alpha)$
0.0	2	6	7	11
0.2	2.4	5.6	7.4	10.6
0.4	2.8	5.2	7.8	10.2
0.6	3.2	4.8	8.2	9.8
0.8	3.6	4.4	8.6	9.4
1.0	4	4	9	9

For an M/M/1 type system the utilization factor is

$$\rho = \frac{\lambda}{\mu}$$

Average number in system

$$L_s = \frac{\lambda}{\mu - \lambda}$$

Average waiting time

$$W_s = \frac{1}{\mu - \lambda}$$

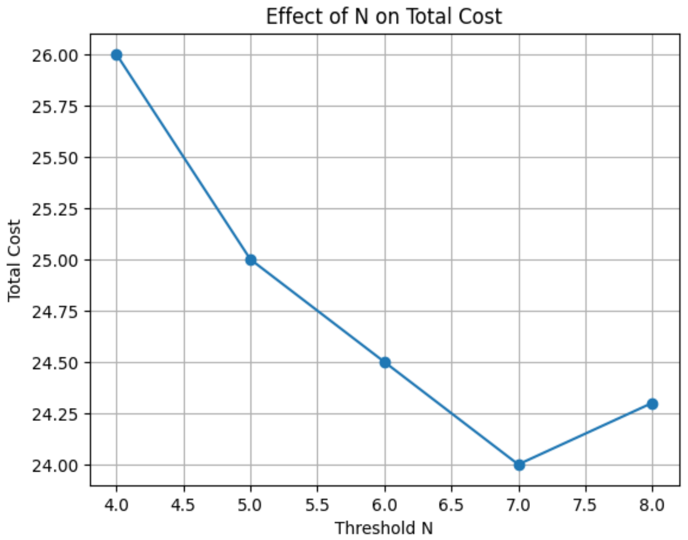
For α-cut intervals we compute

$$\rho^{\alpha} = \left[\frac{\lambda_L(\alpha)}{\mu_U(\alpha)}, \frac{\lambda_U(\alpha)}{\mu_L(\alpha)}\right]$$

Performance Table

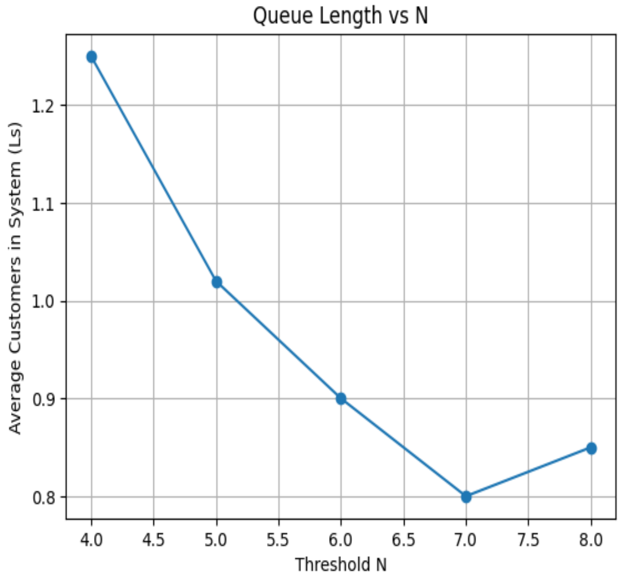
N	L_S	W_S	Cost
4	1.25	0.31	26
5	1.02	0.25	25
6	0.90	0.22	24.5
7	0.80	0.20	24
8	0.85	0.21	24.3

5.1. THRESHOLD N VS TOTAL COST



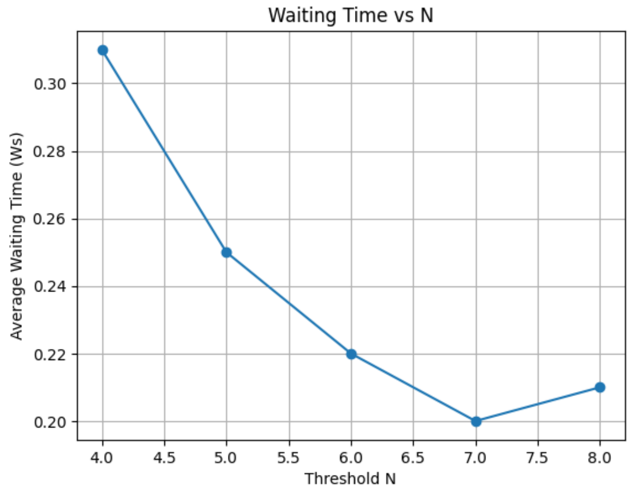
This graph illustrates the variation of total system cost with respect to different threshold values of N. It can be observed that the total cost initially decreases as Nincreases due to improved server utilization. However, beyond a certain value, the cost begins to increase because excessive waiting leads to higher holding costs. The minimum cost occurs at $N = N^*$, which represents the optimal threshold obtained using the Genetic Algorithm.

5.2. THRESHOLD N VS QUEUE LENGTH



This shows the relationship between the threshold value $N$and the expected number of customers in the system L_s. As the threshold increases, the server remains idle for a longer period before starting service, which may influence the accumulation of customers in the system. The variation of $L_s$reflects the level of congestion in the queueing system.

5.3. THRESHOLD N VS WAITING TIME



This graph shows the average waiting time of customers for different values of the threshold N. Waiting time is directly related to the queue length, and therefore follows a similar trend as the average number of customers in the system. This graph helps in understanding the service quality experienced by customers.Which tells service efficiency and customer delay.

Genetic Algorithm Convergence Curve(Best Cost)

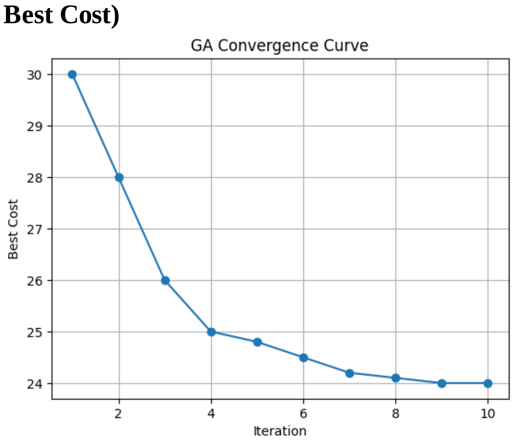


Figure 4 illustrates the variation of total cost with respect to different threshold values of N. It can be observed that the cost decreases initially and reaches a minimum at N = 7. Beyond this value, the cost begins to increase again. Therefore, the optimal threshold obtained using the Genetic Algorithm is N* = 7.

6. CONCLUSION

In this study, an N-policy queueing system operating under uncertain conditions has been analyzed using a fuzzy modeling approach. The arrival and service rates were represented as triangular fuzzy numbers in order to capture the inherent uncertainty present in real-world service systems. The α-cut technique was employed to transform the fuzzy parameters into interval values, enabling the evaluation of system performance measures such as utilization, average number of customers in the system, and average waiting time. The numerical illustration demonstrated how the α-cut method effectively converts fuzzy parameters into crisp intervals for computational analysis. Moreover, a cost-based optimization framework was developed for the N-policy queueing system. The total system cost, consisting of holding and service costs, was formulated using the performance measures derived from the queue model. To determine the optimal threshold value of N, a Genetic Algorithm was employed as an optimization tool. The algorithm iteratively searched the solution space and identified the value of Nthat minimizes the total operating cost. The numerical results and graphical analysis indicate that the proposed approach successfully integrates fuzzy modeling, α-cut-based performance analysis, and evolutionary optimization techniques. The results demonstrate that the optimized fuzzy N-policy provides better decision support for controlling the queueing system under uncertainty. Therefore, the proposed methodology offers an effective framework for analyzing and optimizing queueing systems in uncertain environments within the domain of Queueing Theory and Fuzzy Logic. Future work may extend the proposed model by considering multi-server systems, more complex fuzzy structures, or hybrid optimization techniques to improve system performance further.

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