International Journal of Mathematical Analysis and Research
ISSN: 3108-2637 | Volume 1 Issue 2, 10-15, October-December 2025
DOI: https://doi.org/10.64137/31082637/IJMAR-V1I2P102

Scholastic Research Publication

Received: 17/10/2025 Revised: 19/10/2025 Accepted: 29/10/2025 Published: 03/11/2025

Original Article

(1, 2)*-Weakly Delta Generalized Beta-Compactness and Connectedness in Bitopological Spaces

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ABSTRACT: *In this chapter, the concepts of (1,2)*-weakly delta generalized beta-compactness and (1,2)*-weakly delta-generalized beta-connectedness are introduced, and some characterizations of (1,2)*-weakly delta-generalized beta-compactness and (1,2)*-weakly delta-generalized beta-connectedness are investigated. Also, their properties are discussed. Further, it is proven that (1,2)*-weakly delta-generalized beta-connectedness is preserved under (1,2)*-weakly delta-generalized beta-irresolute mappings.*

KEYWORDS: $(1,2)^*$-*$w\delta g\beta$-compactness,* $(1,2)^*$-*$w\delta g\beta$-connectedness and* $(1,2)^*$-*$w\delta g\beta$-irresolute map.*
AMS Classification: 54C08, 54E55, 54D30

1. INTRODUCTION

In general topology as well as other sophisticated areas of mathematics, the concepts of compactness and connectedness are helpful and essential. Pervin [5] was first to define connectedness and components in a bitopological spaces, whereas the concept of quasi components in bitopological spaces was introduced by Reilly and Young [4]. The generalized closed set has been first studied and initiated by N.Levine in the year 1970 [1]. This generalized closed set has lead to significant contributions to generalization of continuity.The fundamental characteristics of connectivity and compactness have been studied by numerous academics.

Vivekananda Dembre and Pankaj B Gavali [7] introduced and studied the properties of W-compactness and W-connectedness in topological spaces and is to give some characterizations of W-compact spaces. The concept of a weaker form of connectedness was also introduced in generalized topological spaces by Tyagi, et al. [6]. The aim of this paper to introduce the concepts of $(1,2)^*$-wδgβ-separated sets are introduced. The ideas of $(1,2)^*$-wδgβ-connectedness and $(1,2)^*$-wδgβ-compactness in bitopological spaces are also introduced and examined.

2. PRELIMINARIES

Definition 2.1 [2]

A subset M of a bitopological space $\mathcal{F}_1$ is called a (1,2)*-weakly $\delta g\beta$-closed (briefly, (1,2)*-w$\delta g\beta$-closed) if (1,2)*-$\beta cl(M) \subseteq G$ whenever $M \subseteq G$ and G is (1,2)*-δg-open in $\mathcal{F}_1$.
The complement of a (1, 2)*-weakly $\delta g\beta$-closed set is called (1,2)*-weakly $\delta g\beta$-open. We denote the set of all (1,2)*-w$\delta g\beta$-closed sets in $\mathcal{F}_1$ by (1, 2)*- w$\delta g\beta$ -$\mathcal{C}(\mathcal{F}_1)$.

Definition 2.2 [2]

Let $\mathcal{F}_1$ and $\mathcal{F}_2$ be two bitopological spaces. A function $\wp: \mathcal{F}_1 \to \mathcal{F}_2$ is called (1,2)*-weakly $\delta g\beta$-continuous (briefly, (1,2)*-$w\delta g\beta$-continuous) if $\wp^{-1}(G)$ is a (1,2)*-$w\delta g\beta$-open set in $\mathcal{F}_1$ for each $\sigma_{1,2}$-open set G of $\mathcal{F}_2$.

Definition 2.4 [3]:

A bitopological space $\mathcal{F}_1$ is called (1,2)*-$T_{w\delta g\beta}$-space if every (1,2)*-$w\delta g\beta$-closed set in it is $\tau_{1,2}$-closed set.

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3. $(1,2)^*$-wδgβ-COMPACTNESS

In this section, we introduce the concept of $(1,2)^*$-$w\delta g\beta$-compactness in bitopological spaces and obtain some of their properties.

Definition 3.1. A collection $\{M_i : i \in I\}$ of $(1,2)^*$-$w\delta g\beta$-open sets in a bitopological space $\mathcal{F}_1$ is called $(1,2)^*$-$w\delta g\beta$-open cover $\mathcal{F}_1$ of a subset M in $\mathcal{F}_1$ if $M \subseteq \cup_{i\in I} \{M_i: i \in I\}$.

Definition 3.2. A bitopological space $\mathcal{F}_1$ is called $(1,2)^*$-$w\delta g\beta$- compact if every $(1,2)^*$-$w\delta g\beta$-open cover of $\mathcal{F}_1$ has a finite subcover of $\mathcal{F}_1$.

Definition 3.3. A subset M of a bitopological space $\mathcal{F}_1$ is called $(1,2)^*$-$w\delta g\beta$-compact relative to $\mathcal{F}_1$ if for every collection $\{M_i: i \in I\}$ of $(1,2)^*$-$w\delta g\beta$-open subsets of $\mathcal{F}_1$ such that $M \subseteq \cup_{i\in I} \{M_i: i \in I\}$, there exist a finite subset I_0 of I such that $M \subseteq \cup_{i\in I_0} \{M_i: i \in I_0\}$.

Definition 3.4. A subset M of a bitopological space $\mathcal{F}_1$ is called $(1,2)^*$-$w\delta g\beta$-compact if M is $(1,2)^*$-$w\delta g\beta$ compact as the subspace of $\mathcal{F}_1$.

Theorem 3.5. Every $(1,2)^*$-$w\delta g\beta$-compact space is $(1,2)^*$-compact.

Proof. Let $\mathcal{F}_1$ be a $(1,2)^*$-$w\delta g\beta$-compact space. Let $\{M_i: i \in I\}$ be an $\tau_{1,2}$-open cover of $\mathcal{F}_1$. Since every $\tau_{1,2}$-open set is $(1,2)^*$-$w\delta g\beta$-open set, $\{M_i: i \in I\}$ is a $(1,2)^*$-$w\delta g\beta$-open cover of $\mathcal{F}_1$. Since $\mathcal{F}_1$ is $(1,2)^*$-$w\delta g\beta$-compact, the $(1,2)^*$-$w\delta g\beta$-open cover $\{M_i: i \in I\}$ of $\mathcal{F}_1$ has a finite subcover say $\{\{M_i: i \in 1,2,3,\dots,n\}$ for $\mathcal{F}_1$. Therefore $\mathcal{F}_1 = \bigcup_{i=1}^{n} M_i$. Thus the $\tau_{1,2}$-open cover $\{M_i: i \in I\}$ of $\mathcal{F}_1$ is a finite subcover for $\mathcal{F}_1$. Hence $\mathcal{F}_1$ is compact.

Theorem 3.6. A $(1,2)^*$-$w\delta g\beta$-closed subset of a $(1,2)^*$-$w\delta g\beta$-compact space is $(1,2)^*$-$w\delta g\beta$-compact relative to $\mathcal{F}_1$.

Proof. Let M be a $(1,2)^*$-$w\delta g\beta$-closed subset of a $(1,2)^*$-$w\delta g\beta$-compact space $\mathcal{F}_1$. Then M^c is $(1,2)^*$-$w\delta g\beta$-open in $\mathcal{F}_1$. Let $S = \{M_i: i \in I\}$ be a $(1,2)^*$-$w\delta g\beta$-open cover of M by $(1,2)^*$-$w\delta g\beta$-open subsets in $\mathcal{F}_1$. Then $S_1 = S \cup M^c$ is a $(1,2)^*$-$w\delta g\beta$-open cover of $\mathcal{F}_1$. That is, $\mathcal{F}_1 = [\cup M_i: i \in I] \cup M^c$. Since $\mathcal{F}_1$ is $(1,2)^*$-$w\delta g\beta$-compact, S_1 is reducible to a finite subcover of $\mathcal{F}_1$, say $\mathcal{F}_1 = M_{i_1} \cup M_{i_2} \cup \dots \cup M_{i_n} \cup M^c$, $M_{i_k} \in S_1$. But M and M^c are disjoint. Hence $\subseteq M_{i_1} \cup M_{i_2} \cup \dots \cup M_{i_n} \in$ S . Therefore $(1,2)^*$-$w\delta g\beta$-open cover S of M contains a finite subcover. Hence M is $(1,2)^*$-$w\delta g\beta$-compact relative to $\mathcal{F}_1$.

Theorem 3.7. Let $\mathscr{p}: \mathcal{F}_1 \to \mathcal{F}_2$ be a onto $(1,2)^*$-$w\delta g\beta$-continuous map. If $\mathcal{F}_1$ is a $(1,2)^*$-$w\delta g\beta$-compact space, then $\mathcal{F}_2$ is $(1,2)^*$-compact.

Theorem 3.8. If a map $\mathscr{p}: \mathcal{F}_1 \to \mathcal{F}_2$ is $(1,2)^*$-$w\delta g\beta$-irresolute and M a subset of $\mathcal{F}_1$. If M is $(1,2)^*$-$w\delta g\beta$-compact in $\mathcal{F}_1$, then the image $\mathscr{p}(M)$ is $(1,2)^*$-$w\delta g\beta$-compact in $\mathcal{F}_2$.

Proof. Let $\mathscr{p}: \mathcal{F}_1 \to \mathcal{F}_2$ be $(1,2)^*$-$w\delta g\beta$-irresolute and $M \subseteq \mathcal{F}_1$ be $(1,2)^*$-$w\delta g\beta$-compact in $\mathcal{F}_1$.

To prove: $\mathscr{p}(M)$ is $(1,2)^*$-$w\delta g\beta$-compact in $\mathcal{F}_2$. Let $\{M_i : i \in I\}$ be a collection of $(1,2)^*$-$w\delta g\beta$-open subsets of $\mathscr{p}(M)$. Then $\mathscr{p}(M) \subseteq \cup \{M_i : i \in I\}$ and $\mathscr{p}^{-1}(M) \in (1,2)^*$-$w\delta g\beta$-open in $\mathcal{F}_1$. Since $\mathscr{p}$ is $(1,2)^*$-$w\delta g\beta$-irresolute, the inverse image of every $(1,2)^*$-$w\delta g\beta$-open set in $\mathcal{F}_2$ is $(1,2)^*$-$w\delta g\beta$-open in $\mathcal{F}_1$. Hence $\mathscr{p}^{-1}(M_i)$ is $(1,2)^*$-$w\delta g\beta$-open in $\mathcal{F}_1$for each $i \in I$. Now $M \subseteq \mathscr{p}^{-1}[\mathscr{p}(M)] \subseteq \mathscr{p}^{-1}(M_i)$. Thus $\{\mathscr{p}^{-1}(M_i): i \in I\}$ is $(1,2)^*$-$w\delta g\beta$-open cover of M in $\mathcal{F}_1$. Since M is $(1,2)^*$-$w\delta g\beta$-compact in $\mathcal{F}_1$, there exist a finite subset I_0 of I such that $M \subseteq \cup \{\mathscr{p}^{-1}(M_i): i \in I_0\}$. Then $\mathscr{p}(M) \subseteq \cup \{M_i : i \in I_0\}$. Therefore $(1,2)^*$-$w\delta g\beta$-open cover of $\mathscr{p}(M)$ has a finite subcover. Hence $(M\ )$ is $(1,2)^*$-$w\delta g\beta$-compact in $\mathcal{F}_2$.

Definition 3.9. A space $\mathcal{F}_1$ is said to be $(1,2)^*$-$w\delta g\beta$-compact space if $M \subseteq \mathcal{F}_1$ which is $(1,2)^*$-$w\delta g\beta$-compact in $\mathcal{F}_1$ is $(1,2)^*$-$w\delta g\beta$-closed.

Theorem 3.10. Let $\mathcal{F}_1$ be a $(1,2)^*$-$w\delta g\beta$-T_2 and $(1,2)^*$-extremally disconnected space. Then Y is $(1,2)^*$-$w\delta g\beta$-compact space.

Proof: Let M be a subset of $\mathcal{F}_1$ which is $(1,2)^*$-$w\delta g\beta$-compact in $\mathcal{F}_1$ and $x \notin M$. Then for each $y \in M$ there exists two disjoint $(1,2)^*$-$w\delta g\beta$-open sets U and V containing x and y respectively, since $\mathcal{F}_1$ is $(1,2)^*$-$w\delta g\beta$-T_2 space. Also since M is $(1,2)^*$-$w\delta g\beta$-compact in $\mathcal{F}_1$, there exists $y_1, y_2, \dots\dots, y_n \in M$ such that $M \subset \bigcup_{i=1}^{n} V_{y_i}$. Let $U = \bigcup_{i=1}^{n} U_{y_i}$. Then U is a $(1,2)^*$-$w\delta g\beta$-open subset of $\mathcal{F}_1$ that contains x and disjoint from M, since $\mathcal{F}_1$ is $(1,2)^*$-extremely disconnected. Thus $x \notin (1,2)^*$-$w\delta g\beta\mathcal{C}(M)$. Hence M is $(1,2)^*$-$w\delta g\beta$-closed in $\mathcal{F}_1$.

Theorem 3.11. A bitopological space $\mathcal{F}_1$ is $(1,2)^*$-$w\delta g\beta$-compact if and only if every family of $(1,2)^*$-$w\delta g\beta$-closed sets of $\mathcal{F}_1$ having finite intersection property has a non-empty intersection.

Proof. Suppose $\mathcal{F}_1$ is $(1,2)^*$-$w\delta g\beta$-compact. Let $\{M_i : i \in I\}$ be a family of $(1,2)^*$-$w\delta g\beta$-closed sets with finite intersection property.

To prove: $\cap_{i\in I} M_i \neq \emptyset$. Suppose $\cap_{i\in I} M_i = \emptyset$. Then $\mathcal{F}_1 - \cap_{i\in I} M_i = \mathcal{F}_1$. This implies $\cup_{i\in I} (\mathcal{F}_1 - M_i) = \mathcal{F}_1$. Therefore the cover $\{(\mathcal{F}_1 - M_i): i \in I\}$ is a $(1,2)^*$-$w\delta g\beta$-open cover of $\mathcal{F}_1$. Since $\mathcal{F}_1$ is $(1,2)^*$-$w\delta g\beta$-compact, the $(1,2)^*$-$w\delta g\beta$-open cover $\{(\mathcal{F}_1 - M_i): i \in I\}$ has a finite subcover say $\{(\mathcal{F}_1 - M_i): i = 1,2,\dots,n\}$. Therefore $\mathcal{F}_1 = \cup_{i=1}^{n}(\mathcal{F}_1 - M_i) \Rightarrow \mathcal{F}_1 = \mathcal{F}_1 - \cap_{i=1}^{n} M_i \Rightarrow \mathcal{F}_1 - \mathcal{F}_1 = \mathcal{F}_1 - [\mathcal{F}_1 - \cap_{i=1}^{n} M_i] \Rightarrow \emptyset = \cap_{i=1}^{n} M_i$. This contradicts the hypothesis. Hence $\cap_{i\in I} M_i \neq \emptyset$.
Conversely, suppose that every family of $(1,2)^*$-$w\delta g\beta$-closed sets of $\mathcal{F}_1$ with finite intersection property has a non-empty intersection.

To prove: $\mathcal{F}_1$ is $(1,2)^*$-$w\delta g\beta$-compact. Suppose $\mathcal{F}_1$ is not $(1,2)^*$-$w\delta g\beta$-compact. Then there exist a $(1,2)^*$-$w\delta g\beta$-open cover of $\mathcal{F}_1$ say $\{M_i : i \in I\}$ has no finite subcover. Therefore for any finite subfamily $\{M_i : i = 1,2,\dots,n\}$ of $\{M_i : i \in I\}$, we have $\cup_{i=1}^{n} M_i \neq \mathcal{F}_1$, which implies that $\mathcal{F}_1 - \cup_{i=1}^{n} M_i \neq \mathcal{F}_1 - \mathcal{F}_1$, which implies $\cap_{i=1}^{n} (\mathcal{F}_1 - M_i) \neq \emptyset$. Therefore the family $\{(\mathcal{F}_1 - M_i) : i \in I\}$ of $(1,2)^*$-$w\delta g\beta$-closed sets has finite intersection property.
Also by assumption, $\cap_{i\in I} (\mathcal{F}_1 - M_i) \neq \emptyset$ which implies $\mathcal{F}_1 - \cup_{i\in I} M_i \neq \emptyset$ so that $\cup_{i\in I} M_i \neq \mathcal{F}_1$. This implies $\{M_i : i \in I\}$ is not a cover of $\mathcal{F}_1$. This contradicts the fact that $\{M_i : i \in I\}$ is a cover for $\mathcal{F}_1$. Therefore a $(1,2)^*$-$w\delta g\beta$-open cover $\{M_i : i \in I\}$ has a finite subcover $\{\{M_i : i = 1,2,\dots,n\}$. Hence $\mathcal{F}_1$ is $(1,2)^*$-$w\delta g\beta$-compact.

Theorem 3.12. The image of a $(1,2)^*$-$w\delta g\beta$-compact space under a surjective $(1,2)^*$-$w\delta g\beta$-irresolute map is $(1,2)^*$-$w\delta g\beta$-compact.
Proof. Let $\wp: \mathcal{F}_1 \rightarrow \mathcal{F}_2$ be a $(1,2)^*$-$w\delta g\beta$-irresolute map from a $(1,2)^*$-compact space $\mathcal{F}_1$ onto a bitopological space $\mathcal{F}_2$. Let $\{M_i : i \in I\}$ be an $(1,2)^*$-$w\delta g\beta$-open cover of $\mathcal{F}_2$. Since $\wp: \mathcal{F}_1 \rightarrow \mathcal{F}_2$ is $(1,2)^*$-$w\delta g\beta$-irresolute, $\{\wp^{-1}(M_i) : i \in I\}$ is a $(1,2)^*$-$w\delta g\beta$-open cover of $\mathcal{F}_1$. Since $\mathcal{F}_1$ is $(1,2)^*$-$w\delta g\beta$-compact, the $(1,2)^*$-$w\delta g\beta$-open cover $\{\wp^{-1}(M_i) : i \in I\}$ of $\mathcal{F}_1$ has a finite subcover say $\{\wp^{-1}(M_i) : i = 1,2,\dots,n\}$. Therefore $\mathcal{F}_1 = \cup_{i=1}^{n} \wp^{-1}(M_i)$ which implies $\wp(\mathcal{F}_1) = \cup_{i=1}^{n} M_i$. That is $\mathcal{F}_2 = \cup_{i=1}^{n} M_i$. Thus $\{M_1, M_2, \dots, M_n\}$ is a finite subcover of $\{M_i : i \in I\}$ for $\mathcal{F}_2$. Hence $\mathcal{F}_2$ is $(1,2)^*$-$w\delta g\beta$-compact.

4. $(1,2)^*$-Wδgβ-SEPARATED SETS

Definition 4.1. Let $\mathcal{F}_1$ be a bitopological space. Two non-empty subsets M and N are said to be $(1,2)^*$-$w\delta g\beta$-separated if and only if $M \cap (1,2)^*$-$w\delta g\beta cl(N) = \phi$ and $(1,2)^*$-$w\delta g\beta cl(M) \cap N = \phi$. i.e., $[M \cap (1,2)^*$-$w\delta g\beta cl(N)] \cup [(1,2)^*$-$w\delta g\beta cl(M) \cap N] = \phi$.

Theorem 4.2. Two $(1,2)^*$-$w\delta g\beta$-separated sets are always disjoint.
Proof: Let M and N be $(1,2)^*$-$w\delta g\beta$-separated sets. Then, we have $M \cap (1,2)^*$-$w\delta g\beta\mathcal{C}(N) = \emptyset$ and $(1,2)^*$-$w\delta g\beta cl(M) \cap N = \emptyset$. Now, $M \cap N \subseteq (1,2)^*$-$w\delta g\beta cl(M) \cap N = \phi$. This implies $M \cap N = \emptyset$. Hence M and N are disjoint.

Theorem 4.3. Let M and N be two $(1,2)^*$-$w\delta g\beta$-separated sets of $\mathcal{F}_1$. If $C \subseteq M$ and $D \subseteq N$, then C and D are $(1,2)^*$-$w\delta g\beta$-separated.
Proof: Let M and N be two $(1,2)^*$-$w\delta g\beta$-separated sets of a bitopological space $\mathcal{F}_1$. Then $M \cap (1,2)^*$-$w\delta g\beta cl(N) = \phi$ and $(1,2)^*$-$w\delta g\beta cl(M) \cap N = \phi$. Let $C \subseteq M$ and $D \subseteq N$. Then, we have $C \cap (1,2)^*$-$w\delta g\beta cl(D) = \phi$ and $(1,2)^*$-$w\delta g\beta cl(C) \cap D = \phi$. Thus C and D are $(1,2)^*$-$w\delta g\beta$-separated.

Theorem 4.4. Two $(1,2)^*$-$w\delta g\beta$-closed subsets of a bitopological space $\mathcal{F}_1$ are $(1,2)^*$-$w\delta g\beta$-separated if and only if they are disjoint.
Proof: Since $(1,2)^*$-$w\delta g\beta$-separated sets are disjoint, $(1,2)^*$-$w\delta g\beta$-closed separated sets are disjoint. Conversely, let M and N be two disjoint $(1,2)^*$-$w\delta g\beta$-closed sets. We have $(1,2)^*$-$w\delta g\beta cl(M) = M$, $(1,2)^*$-$w\delta g\beta cl(N) = N$ and $M \cap N = \emptyset$. Consequently, $M \cap (1,2)^*$-$w\delta g\beta cl(N) = \phi$ and $(1,2)^*$-$w\delta g\beta cl(M) \cap N = \phi$. Hence M and N are $(1,2)^*$-$w\delta g\beta$-separated.

Theorem 4.5. If the union of two $(1,2)^*$-$w\delta g\beta$-separated sets in a $(1,2)^*$-$w\delta g\beta$-closed set, then the individual sets are $(1,2)^*$-$w\delta g\beta$-closures of themselves.
Proof: Let M and N be two $(1,2)^*$-$w\delta g\beta$-separated sets such that $M \cup N$ is $(1,2)^*$-$w\delta g\beta$-closed. Then we have $M \cap (1,2)^*$-$w\delta g\beta cl(N) = \phi$ and $(1,2)^*$-$w\delta g\beta cl(M) \cap N = \phi$. Also, $M \cup N = (1,2)^* - w\delta g\beta cl(M \cup N)$.

$$= (1,2)^* - w\delta g\beta cl(M) \cup (1,2)^* - w\delta g\beta cl(N). \text{ Therefore}$$

$$\begin{aligned}(1,2)^*\text{-}w\delta g\beta cl(M) &= (1,2)^*\text{-}\, w\delta g\beta cl(M) \cap [(1,2)^*\text{-}\, w\delta g\beta cl(M) \cup (1,2)^*\text{-}w\delta g\beta cl(N)] \\ &= (1,2)^*\text{-}\, w\delta g\beta cl(M) \cap [M \cup N] \\ &= [(1,2)^*\text{-}\, w\delta g\beta cl(M) \cap M] \cup [(1,2)^*\text{-}\, w\delta g\beta cl(M) \cap N]\end{aligned}$$

$= M \cup \phi = M.$

Therefore, $(1,2)^*$-$w\delta g\beta cl(M) = M$. Similarly, $(1,2)^*$-$w\delta g\beta cl(N) = N$.

Hence the proof.

Theorem 4.6. Two disjoint sets M and N be two $(1,2)^*$-$w\delta g\beta$-separated in $\mathcal{F}_1$ if and only if they are both $(1,2)^*$-$w\delta g\beta$-open and $(1,2)^*$-$w\delta g\beta$-closed in $M \cup N$.

Proof: Let M and N be disjoint and $(1,2)^*$-$w\delta g\beta$-separated in $\mathcal{F}_1$. Then, $M \cap (1,2)^*$-$w\delta g\beta cl(N) = \phi$ and $(1,2)^*$-$w\delta g\beta cl(M) \cap N = \phi$. Let $E = M \cup N$. Then, $(1,2)^*$-$w\delta g\beta cl_E(M) = (1,2)^*$-$w\delta g\beta cl(M) \cap E = (1,2)^*$-$w\delta g\beta cl(M) \cap (M \cup N) = M$. Therefore M is $(1,2)^*$-$w\delta g\beta$-closed in E. Similarly, N is $(1,2)^*$-$w\delta g\beta$-closed in E. Now, $M \cap N = \emptyset$ and $M \cup N = E$. So M and N are complements of each other in E. Thus each one of M and N is $(1,2)^*$-$w\delta g\beta$-open in E. Conversely, let M and N be disjoint sets which are both $(1,2)^*$-$w\delta g\beta$-open and $(1,2)^*$-$w\delta g\beta$-closed in $E = M \cup N$.

Then $M = (1,2)^*$-$w\delta g\beta cl_E(M) = (1,2)^*$-$w\delta g\beta cl(M) \cap E$

$= (1,2)^*$-$w\delta g\beta cl(M) \cap (M \cup N)$

$= M \cup [(1,2)^*$-$w\delta g\beta cl(M) \cap N]$

But $\cap N = \phi$. This implies $(1,2)^*$-$w\delta g\beta cl(M) \cap N = \emptyset$.

Now, M and $(1,2)^*$-$w\delta g\beta cl(M) \cap N$ are disjoint and their union is M. So, $(1,2)^*$-$w\delta g\beta cl(M) \cap N = \phi$. Similarly, $M \cap (1,2)^*$-$w\delta g\beta cl(N) = \phi$. Hence M and N are $(1,2)^*$-$w\delta g\beta$-separated.

5.$(1,2)^*$-Wδgβ-CONNECTEDNESS

Definition 5.1. A bitopological space $\mathcal{F}_1$ is said to be $(1,2)^*$-$w\delta g\beta$-connected if $\mathcal{F}_1$ cannot be expressed as a disjoint union of two non-empty $(1,2)^*$-$w\delta g\beta$-open sets.

A subset of $\mathcal{F}_1$ is $(1,2)^*$-$w\delta g\beta$-connected if it is $(1,2)^*$-$w\delta g\beta$-connected as a subspace.

Example 5.2. Let $\mathcal{F}_1 = \{a,b,c\}$. Let $\tau_1 = \{\emptyset, \mathcal{F}_1, \{b\}\}$, $\tau_1 = \{\emptyset, \mathcal{F}_1, \{a,b\}\}$ and $\tau_{1,2} = \{\emptyset, \mathcal{F}_1, \{b\}, \{a,b\}\}$. The $(1,2)^*$-$w\delta g\beta$-open sets of $\mathcal{F}_1$ are $\emptyset, \{b\}, \{a,b\}, \{b,c\}, \mathcal{F}_1$. Then $\mathcal{F}_1$ is a $(1,2)^*$-$w\delta g\beta$- connected space.

Remark 5.3. Every $(1,2)^*$-$w\delta g\beta$-connected space is $(1,2)^*$-connected but the converse need not be true which follows from the following example.

Example 5.4. Let $\mathcal{F}_1 = \{a,b,c\}$. Let $\tau_1 = \{\emptyset, \mathcal{F}_1, \{a\}\}$, $\tau_1 = \{\emptyset, \mathcal{F}_1, \{c\}, \{a,c\}\}$ and $\tau_{1,2} = \{\emptyset, \mathcal{F}_1, \{a\}, \{c\}, \{a,c\}\}$. $(1,2)^*$-$w\delta g\beta$-open sets of $\mathcal{F}_1$ are $\emptyset, \{a\}, \{b\}, \{a,b\}, \{a,c\}, \{b,c\}, \mathcal{F}_1$. Since $\mathcal{F}_1 = \{a\} \cup \{\mathrm{b,c}\}$, where $\{a\}$ and $\{\mathrm{b,c}\}$ are disjoint non empty $(1,2)^*$-$w\delta g\beta$-open sets, hence $\mathcal{F}_1$ is not a $(1,2)^*$-$w\delta g\beta$-connected space.

Definition 5.5. A map $\wp : \mathcal{F}_1 \to \mathcal{F}_2$ is said to be strongly $(1,2)^*$-$w\delta g\beta$-closed if for each $(1,2)^*$-$w\delta g\beta$-closed set G of $\mathcal{F}_1$, $\wp(G)$ is $(1,2)^*$-$w\delta g\beta$-closed set in $\mathcal{F}_2$.

Definition 5.6. A map $\wp : \mathcal{F}_1 \to \mathcal{F}_2$ is said to be quasi-$(1,2)^*$-$w\delta g\beta$-closed if for each $(1,2)^*$-$w\delta g\beta$-closed set G of $\mathcal{F}_1$, $\wp(G)$ is closed set in $\mathcal{F}_2$.

Theorem.5.7. For a bitopological space $\mathcal{F}_1$, the following are equivalent.

(i). $\mathcal{F}_1$ is $(1,2)^*$-$w\delta g\beta$-connected.

(ii). $\mathcal{F}_1$ and $\emptyset$ are the only subsets of $\mathcal{F}_1$ which are both $(1,2)^*$-$w\delta g\beta$-open and $(1,2)^*$-$w\delta g\beta$-closed.

(iii). Each $(1,2)^*$-$w\delta g\beta$-continuous map of $\mathcal{F}_1$ into a discrete space $\mathcal{F}_2$ with atleast two points is a constant map.

Proof: (i)⇒(ii). Let $M \subseteq \mathcal{F}_1$ be any proper subset, which is both $(1,2)^*$-$w\delta g\beta$-open and $(1,2)^*$-$w\delta g\beta$-closed. Its complement $\mathcal{F}_1 - M$ is also $(1,2)^*$-$w\delta g\beta$-open and $(1,2)^*$-$w\delta g\beta$-closed. Then $\mathcal{F}_1 = M \cup (\mathcal{F}_1 - M)$ is a disjoint union of two non-empty $(1,2)^*$-$w\delta g\beta$-open sets which is a contradiction that $\mathcal{F}_1$ is $(1,2)^*$-$w\delta g\beta$-connected. Hence, either $M = \emptyset$ or $\mathcal{F}_1$.

$(ii) \Rightarrow (i)$. Let $\mathcal{F}_1 = G \cup H$ here $\cap H = \emptyset$, $G \neq \emptyset$, $H \neq \emptyset$ and G, H are $(1,2)^*$-$w\delta g\beta$- open. Since $G = \mathcal{F}_1 - H$, G is $(1,2)^*$-$w\delta g\beta$-closed. According to the assumption $G = \emptyset$, which is a contradiction. Hence $\mathcal{F}_1$ is $(1,2)^*$-$w\delta g\beta$-connected.

(ii)⇒(iii). Let $\wp : \mathcal{F}_1 \to \mathcal{F}_2$ be a $(1,2)^*$-$w\delta g\beta$-continuous function where $\mathcal{F}_2$ is a discrete bitopological space with at least two points. Then $\wp^{-1}(\{m\})$ is $(1,2)^*$-$w\delta g\beta$-closed and $(1,2)^*$-$w\delta g\beta$-open for each $m \in \mathcal{F}_2$ and $\mathcal{F}_1 = \cup \{\wp^{-1}(\{m\}) : m \in \mathcal{F}_2\}$. According to the assumption, $\wp^{-1}(\{m\}) = \emptyset$ or $\mathcal{F}_1$. If $\wp^{-1}(\{m\}) = \emptyset$ for all $m \in \mathcal{F}_2$, $\wp$ will not be a function. Also there exist atleast one $m_1 \in \mathcal{F}_2$, $\wp^{-1}(\{m_1\}) \neq \emptyset$ such that $\wp^{-1}(\{m_1\}) = \mathcal{F}_1$. This shows that $\wp$ is a constant function.

(iii)⇒(ii). Let $M \neq \emptyset$ be both $(1,2)^*$-$w\delta g\beta$-open and $(1,2)^*$-$w\delta g\beta$-closed in $\mathcal{F}_1$. Let $\wp : \mathcal{F}_1 \to \mathcal{F}_2$ be a $(1,2)^*$-$w\delta g\beta$-continuous function defined by $\wp(M) = \{m\}$ and $\wp(\mathcal{F}_1 - M) = \{n\}$ where $m \neq n$. Since $\wp$ is constant, we get $M = \mathcal{F}_1$.

Theorem 5.8. If $\wp : \mathcal{F}_1 \to \mathcal{F}_2$ is a $(1,2)^*$-$w\delta g\beta$-continuous surjection map and $\mathcal{F}_1$ is $(1,2)^*$-$w\delta g\beta$-connected then $\mathcal{F}_2$ is $(1,2)^*$-connected.

Proof. Suppose that $\mathcal{F}_2$ is not $(1,2)^*$-connected.
Then $\mathcal{F}_2 = M \cup N$ where M and N are disjoint non-empty $\sigma_{1,2}$-open sets in $\mathcal{F}_2$. Since $\wp$ is $(1,2)^*$-$w\delta g\beta$-continuous and onto, $\mathcal{F}_1 = \wp^{-1}(M) \cup \wp^{-1}(N)$ where $\wp^{-1}(M)$ and $\wp^{-1}(N)$ are disjoint non-empty $(1,2)^*$-$w\delta g\beta$-open sets in $\mathcal{F}_1$. This contradicts the fact that $\mathcal{F}_1$ is $(1,2)^*$-$w\delta g\beta$-connected. Hence $\mathcal{F}_2$ is $(1,2)^*$-connected.

Theorem 5.9. If $\wp : \mathcal{F}_1 \to \mathcal{F}_2$ is a $(1,2)^*$-$w\delta g\beta$-irresolute surjection and $\mathcal{F}_1$ is $(1,2)^*$-$w\delta g\beta$-connected then $\mathcal{F}_2$ is $(1,2)^*$-$w\delta g\beta$-connected.

Proof. Suppose that $\mathcal{F}_2$ is not $(1,2)^*$-$w\delta g\beta$-connected. Then $\mathcal{F}_2 = M \cup N$ where M and N are disjoint non-empty $(1,2)^*$-$w\delta g\beta$-open sets in $\mathcal{F}_2$. Since $\wp$ is $(1,2)^*$-$w\delta g\beta$-irresolute and onto, $\mathcal{F}_1 = \wp^{-1}(M) \cup f^{-1}(N)$ where and $f^{-1}(M)$ are $f^{-1}(N)$ disjoint non-empty $(1,2)^*$-$w\delta g\beta$-open sets in $\mathcal{F}_1$. This contradicts the fact that $\mathcal{F}_1$ is $(1,2)^*$-$w\delta g\beta$-connected. Hence $\mathcal{F}_2$ is $(1,2)^*$-$w\delta g\beta$-connected.

Theorem 5.10. $\mathcal{F}_1$ is $(1,2)^*$-connected if and only if $(1,2)^*$-$w\delta g\beta$-connected when $\mathcal{F}_1$ is $(1,2)^*$-$T_{w\delta g\beta}$-Space.

Proof. Suppose that $\mathcal{F}_1$ is $(1,2)^*$-connected. Then $\mathcal{F}_1$ cannot be expressed as disjoint union of two non-empty proper subsets of $\mathcal{F}_1$. Suppose $\mathcal{F}_1$ is not a $(1,2)^* w\delta g\beta$-connected space. Let M and N be any two $(1,2)^*$-$w\delta g\beta$ open subsets of $\mathcal{F}_1$ such that $\mathcal{F}_1 = M \cup N$, where $M \cap N = \emptyset$ and $M \subset \mathcal{F}_1$, $N \subset \mathcal{F}_1$. Since $\mathcal{F}_1$ is $(1,2)^*$-$T_{w\delta g\beta}$-space and M, N are $(1,2)^*$-$\delta g\beta$-open, M, N are $\tau_{1,2}$-open subsets of $\mathcal{F}_1$. This contradicts the fact that $\mathcal{F}_1$ is $(1,2)^*$-connected. Therefore $\mathcal{F}_1$ is $(1,2)^*$-$w\delta g\beta$- connected. Conversely, Since every $\tau_{1,2}$-open set is $(1,2)^*$-$w\delta g\beta$-open, every $(1,2)^*$-$w\delta g\beta$-connected space is $(1,2)^*$-connected.

Theorem 5.11. If the sets M and N form a $(1,2)^*$-$w\delta g\beta$-separation of $\mathcal{F}_1$ and if $\mathcal{F}_2$ is $(1,2)^*$-$w\delta g\beta$-connected subspace of $\mathcal{F}_1$ then $\mathcal{F}_2$ lies entirely within M or N.

Proof. Since M and N are both $(1,2)^*$-$w\delta g\beta$-open sets in $\mathcal{F}_1$, the sets $M \cap \mathcal{F}_2$ and $N \cap \mathcal{F}_2$ are $(1,2)^*$-$w\delta g\beta$-open in $\mathcal{F}_2$, these two sets are disjoint and their union is $\mathcal{F}_2$. If they were both non-empty, they would constitute a $(1,2)^*$-$w\delta g\beta$-separation of $\mathcal{F}_2$. Therefore one of them is empty. Hence $\mathcal{F}_2$ must lie entirely in M or in N.

Theorem 5.12. If $\wp : \mathcal{F}_1 \to \mathcal{F}_2$ is a bijective $(1,2)^*$-$w\delta g\beta$-closed map of a space $\mathcal{F}_1$ onto a $(1,2)^*$-$w\delta g\beta$-connected space $\mathcal{F}_2$, then $\mathcal{F}_1$ is $(1,2)^*$-connected.

Proof. Assume that $\mathcal{F}_1$ is not $(1,2)^*$-connected. Then there exist non empty $\tau_{1,2}$-open sets M and N such that $M \cap N = \emptyset$ and $\mathcal{F}_1 = M \cup N$. Since $\wp : \mathcal{F}_1 \to \mathcal{F}_2$ is $(1,2)^*$-$w\delta g\beta$-closed map, we have $\wp(M^c)$ and $p(N^c)$ are $(1,2)^*$-$w\delta g\beta$-closed sets in $\mathcal{F}_2$. Now $\wp(M^c) = [\wp(M)]^c$ and $\wp(N^c) = [\wp(N)]^c$. Then $\wp(M)$ and $\wp(N)$ are $(1,2)^*$-$w\delta g\beta$-open sets in $\mathcal{F}_2$.
Also $\wp(M) \cap \wp(N) = \emptyset$ and $\wp(M) \cup \wp(N) = \mathcal{F}_2$. Since p is bijective, $\wp(M)$ and $\wp(N)$ are non empty. This implies $\mathcal{F}_2$ is not $(1,2)^*$-connected. This is a contradiction. Hence $\mathcal{F}_1$ is $(1,2)^*$-connected.

Theorem 5.13. Let $\wp : \mathcal{F}_1 \to \mathcal{F}_2$ be a bijective map. Then the following results hold.

(i). If $\wp$ is a strongly $(1,2)^*$-$w\delta g\beta$-closed map and $\mathcal{F}_2$ is a $(1,2)^*$-$w\delta g\beta$-connected space, then $\mathcal{F}_1$ is $(1,2)^*$-$w\delta g\beta$ connected.

(ii). If $\wp$ is a quasi-$(1,2)^*$-$w\delta g\beta$-closed map and $\mathcal{F}_2$ is a connected space, then $\mathcal{F}_1$ is $(1,2)^*$-$\delta g\beta$-connected.

Proof. Let $\wp : \mathcal{F}_1 \to \mathcal{F}_2$ be a bijective map.

(i). Suppose p is a strongly $(1,2)^*$-$w\delta g\beta$-closed map and $\mathcal{F}_2$ is a $(1,2)^*$-$w\delta g\beta$-connected space. Assume that $\mathcal{F}_1$ is not $(1,2)^*$-$w\delta g\beta$-connected space. Then there exist non empty $(1,2)^*$-$w\delta g\beta$-open sets M and N such that $M \cap N = \emptyset$ and $\mathcal{F}_1 = M \cup N$.
Since $\wp : \mathcal{F}_1 \to \mathcal{F}_2$ is strongly $(1,2)^*$-$w\delta g\beta$-map, we have $\wp(M^c)$ and $\wp(N^c)$ are $(1,2)^*$-$w\delta g\beta$-closed sets in $\mathcal{F}_2$. Now $\wp(M^c) = [\wp(M)]^c$ and $\wp(N^c) = [\wp(N)]^c$.
Then $\wp(M)$ and $p(N)$ are $(1,2)^*$-$w\delta g\beta$-open sets in $\mathcal{F}_2$.
Also $\wp(M) \cap p(N) = \emptyset$ and $\wp(M) \cup \wp(N) = \mathcal{F}_2$.
Since $\wp$ is bijective, $\wp(M)$ and $\wp(N)$ are non empty.
This implies $\mathcal{F}_2$ is not $(1,2)^*$-$w\delta g\beta$-connected. This is a contradiction.
Hence $\mathcal{F}_1$ is a $(1,2)^*$-$w\delta g\beta$-connected space.

(ii) Suppose $\wp$ is a quasi $(1,2)^*$-$w\delta g\beta$-closed map and $\mathcal{F}_2$ is a $(1,2)^*$-connected space.
Assume that $\mathcal{F}_1$ is not $(1,2)^*$-$w\delta g\beta$-connected space. Then there exist non empty $(1,2)^*$- $w\delta g\beta$-open sets M and Nsuch that $M \cap N = \emptyset$ and $\mathcal{F}_1 = M \cup N$. Since $\wp : \mathcal{F}_1 \to \mathcal{F}_2$ is quasi $(1,2)^*$-$w\delta g\beta$-closed map, we have $\wp(U^c)$ and $\wp(V^c)$ are

$\sigma_{1,2}$-closed sets in $\mathcal{F}_2$.
Now $\wp(M^c) = [\wp(M)]^c$ and $\wp(N^c) = [\wp(N)]^c$. Then $\wp(M)$ and $\wp(N)$ are open sets in $\mathcal{F}_2$.
Also $\wp(M) \cap \wp(N) = \emptyset$ and $\wp(U) \cup \wp(V) = \mathcal{F}_2$. Since $\wp$ is bijective, $\wp(M)$ and $\wp(N)$ are non empty.
This implies $\mathcal{F}_2$ is not $(1,2)^*$-connected. This is a contradiction.
Hence $\mathcal{F}_1$ is a $(1,2)^*$-$w\delta g\beta$-connected space.

Theorem 5.14. A bitopological space $\mathcal{F}_1$ is $(1,2)^*$-$w\delta g\beta$-disconnected if there exists a non-empty proper subset of $\mathcal{F}_1$ which is both $(1,2)^*$-$w\delta g\beta$-open and $(1,2)^*$-$w\delta g\beta$-closed.
Proof: Let M be a non-empty proper subset of $\mathcal{F}_1$ which is both $(1,2)^*$-$w\delta g\beta$-open and $(1,2)^*$-$w\delta g\beta$-closed.
Then clearly M^c is a non-empty proper subset of $\mathcal{F}_1$ which is both $(1,2)^*$-$w\delta g\beta$-open and $(1,2)^*$-$w\delta g\beta$-closed.
Thus $M \cap M^c = \phi$, therefore $M \cap\ (1,2)^* - w\delta g\beta - cl(M^c\ ) = \phi$ and $(1,2)^* - w\delta g\beta - cl(M^c) \cap M = \phi$.
Also, $\mathcal{F}_1 = M \cup M^c$. Thus $\mathcal{F}_1$ is the union of two non-empty $(1,2)^*$-$w\delta g\beta$-separated sets.
Hence $\mathcal{F}_1$ is $(1,2)^*$-$w\delta g\beta$-disconnected.

Theorem 5.15. If every two points of a set M are contained in some $(1,2)^*$-$w\delta g\beta$-connected subset of M, then M is $(1,2)^*$-$w\delta g\beta$-connected.
Proof: Suppose that M is not $(1,2)^*$-$w\delta g\beta$-connected.
Then M is the union of two non-empty disjoint $(1,2)^*$-$w\delta g\beta$-open sets G and H.
Let g $g{\in}G$ and $h{\in}H$. Then g and h are two distinct points of M.
By hypothesis, there exists a $(1,2)^*$-$w\delta g\beta$-connected subset N of M such that $g, h\ \in N$.
But N is an $(1,2)^*$-$w\delta g\beta$-connected subset of a $(1,2)^*$-$w\delta g\beta$-disconnected set M, we have $N \subseteq$ G or $N \subseteq$ H.
Since G and H are disjoint and N contains atleast one point of G and one of that of H.
This is a contradiction. Hence M is $(1,2)^*$-$w\delta g\beta$-connected.
Proof: Assume that $g_1 \in (1,2)^*$-$w\delta g\beta - int(M)$. There is an $(1,2)^*$-$w\delta g\beta$-open set V such that $g_1 \in V \subset M$.
For every $g_2 \in M$, we have $g_2 * V = g_2 * {g_1}^{-1} * g_1 * g_1 \subset g_2 * {g_1}^{-1} * M = M$.
Since $g_2 * V$ is $(1,2)^*$-$w\delta g\beta$-open set, then $M = \cup \{g_2 * V : g_2 \in M\}$ is $(1,2)^*$-$w\delta g\beta$-open as the union of $(1,2)^*$-$w\delta g\beta$-open sets. The converse is trivial.

6. CONCLUSION

In this chapter, the concepts of $(1,2)^*$-$w\delta g\beta$ compactness and $(1,2)^*$-$w\delta g\beta$ -connectedness in bitopological spaces were introduced and studied. Several characterizations of these notions were established, providing deeper insight into their structural properties. The fundamental properties of $(1,2)^*$-wδgβ-compact and $(1,2)^*$-$w\delta g\beta$ -connected spaces were also discussed. It was shown that $(1,2)^*$-$w\delta g\beta$ -connectedness is preserved under $(1,2)^*$-$w\delta g\beta$ -irresolute mappings, which demonstrates the stability of this concept under suitable functions between bitopological spaces.

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