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Original Article

Edge Total Mean Labeling of Graphs

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ABSTRACT: *In this paper, we introduce a new labeling edge total mean labeling. An edge total mean labeling f: V ∪ E → {1, 2, ..., p + q} of a graph G = G(V, E) is a labeling of vertices and edges of a graph in such a way that for any two different edges uv and u′ v′ their mean* $m(uv) = \frac{f(u)+f(uv)+f(v)}{3}$ *and* $m(u'v') = \frac{f(u')+f(u'v')+f(v')}{3}$ *are distinct and the result in edge total means varies from 1,2,. . . ,q. A graph G is edge total mean graph if it admits edge total mean labeling. In this paper, we introduce a concept of edge total mean labeling of some graphs.*

KEYWORDS: *Mean labeling, Total labeling, Total mean labeling, Edge total mean labeling, Cycle, Wheel, Helm, Closed helm, Double wheel, Fan, Double fan, Gear, Sun and flower graphs.*

AMS Subject Classification: 05C78

1. INTRODUCTION

As a standard notation, assume that G = G (V, E) is a finite, simple and undirected graph with p vertices and q edges. Terms and terminology we follow as in Harary [2]. Graph labeling was first introduced by Alex Rosa in 1967. During the last five decades nearly 300 graph labeling techniques have been studied in over 2800 papers. A labeling of a graph is a map that carries graph elements to the numbers (usually to the positive or non-negative integers). The most common choices of domain are the set of all vertices (vertex labeling), the set of edge (edge labeling), or the set of all vertices and edges (total labeling). In 2003, Somasundaram and Ponraj [10] introduced the notion of mean labeling of graphs. Let G be a (p, q) graph. A graph G is called a mean graph if there is an injective function f from the vertices of G to {0, 1, 2, ..., q} such that when each edge uv is labeled with $\frac{f(u)+f(v)}{2}$ if f (u) + f (v) is even, and $\frac{f(u)+f(v)+1}{2}$ if f (u) + f (v) is odd, then the resulting edge labels are distinct. Several results have been published by several authors on mean labeling and its variations. An excellent survey of graph labeling is available in [1]. With the growing interest on the variations of the mean labelling, we naturally define edge total mean labeling as follows:

An edge total mean labeling f : V ∪ E −→ {1, 2, . . . , p + q} of a graph G = G(V, E) is a labeling of vertices and edges of G in such a way that for any two different edges uv and u'v', their mean $m(uv) = \frac{f(u)+f(v)+1}{3}$ and $m(u'v') = \frac{f(u')+f(u'v')+f(v')}{3}$ are distinct and the resulting edge total means varies from 1,2,. . . ,q. For integers a < b, we let [a, b] = {a, a + 1, . . . , b}.

The following definitions are used in the subsequent section to prove our main results.

Definition 1.1. [9] A bijection f : V ∪ E → {1, 2, . . . , p + q} is said to be a total mean labeling if for each uv ∈ E(G), $f*(uv) = [\frac{f(u)+f(v)+f(uv)}{3}]$ is distinct. A graph G is said to be a total mean labeling graph if it admits a total mean graph.

Definition 1.2. [6] A corona product of two graphs G and H, denoted by G⨀H, is the graph that is obtained by placing a copy of G and |V (G)| copies of H so that all vertices in the same copy of H are joined with exactly one vertex of G, while each vertex of G is joined to exactly one copy of H.

Definition 1.3. [3] The helm graph H_n is obtained from a wheel by attaching a pendant edge at each vertex of the n-cycle. Thus the vertex set of Hn is V (H_n) = {u, u_i, u_i : 1 ≤ i ≤ n} and the edge set of H_n is E(H_n) = {uu_i, u_iu_i+1, u_iu_i : 1 ≤ i ≤ n} with indices taken modulo n.

Definition 1.4. [3] The closed helm graph CH_n is obtained from a helm H_n by joining each pendant vertex to form a cycle. It contains three types of vertices: an apex of degree n, n vertices of degree 4 and n vertices of degree 3. Thus the vertex set of

H_n is V (CH_n) = {u_i, u_i, v: 1 ≤ i ≤ n} and the edge set is E(CH_n) = {uu_i, u_iu_i+1, u_iu_i, u_iu_i+1 : 1 ≤ i ≤ n} with indices taken modulo n.

Definition 1.5. [1] The fan graph F_n can be constructed from a wheel by deleting one edge in C_n. Thus the vertex set of F_n is V (F_n) = {v, v_i : 1 ≤ i ≤ n} and the edge set of F_n is E(F^n) = {uu_i : 1 ≤ i ≤ n} ∪ {u_iu_i+1 : 1 ≤ i ≤ n − 1}.

Definition 1.6. [4] The graph $P_n + 2K_1$ is called a double fan graph DF_n.

Definition 1.7. [3] The gear graph G_n is obtained from the wheel W_n by subdividing each of its rim edge. It is also known as The Jahangir graph $J_{n,2}$.Thus the vertex set of G_n is V (G_n) = {u, u_i, u_i : 1 ≤ i ≤ n} and the edge set of G_n is E(G_n) = {uu_i, u_iu_i, u_iu_i+1 : 1 ≤ i ≤ n}, with indices taken modulo n.

Definition 1.8. [1]The sun graph S_n, is a graph on 2_n vertices obtained by attaching n-pendant edges to the cycle Cn.

Definition 1.9. [3]The f lower graph Fl_n is obtained from a helm by joining each pendant vertex to the central vertex of the helm. Thus the vertex set of Fl_n is V (Fl_n) = {u, u_i, u_i : 1 ≤ i ≤ n} and the edge set of Fl_n is E(Fl_n) = {uu_i, uu_i, u_iu_i+1, u_i u_i : 1 ≤ i ≤ n}, with indices taken modulo n.

2. MAIN RESULTS

In this section, we establish that the graphs helm, closed helm, double wheel, fan, double fan, gear, sun and flower graphs admit edge total mean labeling.

Theorem 2.1. *A graph G=(V,E) be a (p,q) graph. If G admits Edge total mean labeling, then the maximum edge total mean labeling of G is q*

Proof. Let A graph G=(V,E) be a (p,q) graph. Suppose G admits Edge total mean labeling, from the definition of edge total mean labeling, means of the edges {1,2,. . . ,q},obviously maximum edge total mean labeling of G is equal to no of edges.

Theorem 2.2. *Let A graph G=(V,E) be a (p,q) graph. Suppose An edge total mean labeling $f : V \cup E \to \{1, 2, . . . , p + q\}$ is bijection .Then G admits total mean labeling that is G is Total mean graph* □

Proof. Let A graph G=(V,E) be a (p,q) graph. If G admits edge total mean labeling, suppose $f : V \cup E \to \{1, 2, . . . , p + q\}$ is a bijection ,since edge total mean graph has distinct means from 1,2,. . . ,q, from the definition of total mean graph, its G admits total mean labeling .Hence G is a total mean graph.

Corollary 2.3. *Every Edge total mean graph is a Total mean graph if f is bijective*

Theorem 2.4. *Every Edge total mean graph is not bijection*

Proof. Let $G = (p, q)$ be an Edge Total mean graph. Since the graph $f : V \cup E \longrightarrow \{1, 2, . . . , p + q\}$admits edge total mean labeling and the resulting means are $\{1, 2, . . . , q\}$. and G has q edges here the maximum means are q. minimum means 1. Since atleast one edge(two vertices) of G must contain the same number 1. Hence it is not objective. □

Theorem 2.5. *The path graph p_n, $n \geq 2$ admits an edge total mean labeling.*

□

Proof. Define $f : V \cup E \longrightarrow [1, 2n - 1]$ as follows:

$$f(u_i) = \begin{cases} 1, & if\ i = 1 \\ \quad i - 1 & if 2 \leq i \leq n; \end{cases}$$

$$f(u_i\, u_i + 1) = \begin{cases} 1, & if\ i = 1 \\ i + 1, & if\ 2 \leq i \leq n - 1 \end{cases}$$

From the definition of f, all vertex and edge labels are at most 2n − 1.The edge total means are m(u_iu_i+1) = i, 1 ≤ i ≤ n.

Hence, the edge total means form the set [1, n − 1]. Therefore, f is an edge total mean labeling. An example of edge total mean labeling of path graph P4 is shown in Figure 1.

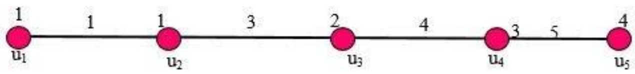

FIGURE 1 **Edge total mean labeling of P_5**

Theorem 2.6. *The corona product of $P_n \odot P_1$ $n \geq 2$ admits an edge total mean labeling.*

Proof. $|V(G)| = 2n$, $|E(G)| = 2n - 1$. Define $f : V \cup E \longrightarrow [1, 4n-1]$ as follows:

$$f(u_i) = \begin{cases} 1, & if\ i = 1 \\ i-1, & if\ i = 2 \leq i \leq n; \end{cases}$$

$$f(v_i) = 3n - 2, 1 \leq i \leq n;$$

$$f(u_i u_i + 1) = \begin{cases} 1, & if\ i = 1 \\ i+1, & if\ 2 \leq i \leq n-1; \end{cases}$$

$$f(u_i u_i) = \begin{cases} 1, & if\ i = 1 \\ 2i, & if\ 2 \leq i \leq n \end{cases}$$

From the definition of *f*, all vertex and edge labels are at most $4n - 1$.The edge total means are
$m(u_i u_i{+}1) = i,\ 1 \leq i \leq n - 1;$
$m(v_i u_i) = n - 1 + i,\ 1 \leq i \leq n;$
$m(uu_i) = 2n + i,\ 1 \leq i \leq n;$

Hence, the edge total means form the set$[1, 2n - 1]$. Therefore, f is an edge total mean labeling. An example of edge total mean labeling of P5⊙P1 is shown in Figure 2. □

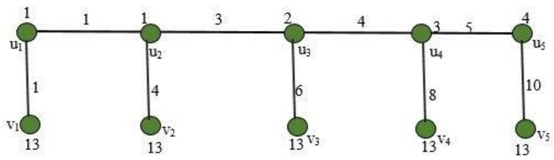

FIGURE 2 **Edge total mean labeling of corona product of P_5 and P_1**

Theorem 2.7. The helm graph Hn, $n \geq 3$ admits an edge total mean labeling.
Proof. Define $f : V \cup E \longrightarrow [1, 5n + 1]$ as follows:

$$f(v) = 5n + 1;$$

$$f(v_i) = \begin{cases} 1, & if\ i = 1 \\ i-1, & if\ 2 \leq i \leq n; \end{cases}$$

$$f(u_i u_i + 1) = \begin{cases} 1, & if\ i = 1 \\ i+1, & if\ 2 \leq i \leq n-1 \\ 2n, & if\ i = n; \end{cases}$$

$$f(u_i) = n - 1 + i, 1 \leq i \leq n;$$

$$f(u_i u_i) = \begin{cases} 2(n+1) & if\ i = 1 \\ 2n + 2 + i, & if\ 2 \leq i \leq n; \end{cases}$$

$$f(uu_i) = \begin{cases} n+1, & if\ i = 1 \\ n + 2i, & if\ 2 \leq i \leq n. \end{cases}$$

From the definition of f , all vertex and edge labels are at most $5n + 1$.The edge total means are
$m(v_i v_i{+}1) = i,\ 1 \leq i \leq n;$
$m(v_i u_i) = n + i,\ 1 \leq i \leq n;$
$m(uu_i) = 2n + i,\ 1 \leq i \leq n;$
Hence, the edge total means form the set$[1, 3n]$. Therefore, f is an edge total mean labeling. An example of edge total mean labeling of helm graph H6 is shown in Figure 3.

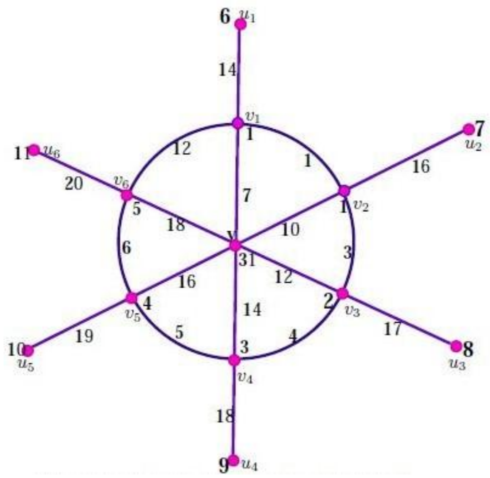

FIGURE 3 **Edge total mean labeling of H_6**

Theorem 2.8. The closed helm graph CHn, n ≥ 3 admits an edge total mean labeling.
Proof. Define f : V ∪ E −→ [1, 6n + 1] as follows:

$$f(v) = 6n + 1;$$

$$f(u_i) = \begin{cases} 1, & if\ i = 1 \\ i - 1, & if\ 2 \leq i \leq n; \end{cases}$$

$$f(u_iu_i + 1) = \begin{cases} 1, & if\ i = 1 \\ i + 1, & if\ 2 \leq i \leq n - 1 \\ 2n, & if\ i = n; \end{cases}$$

$$f(u_i) = n - 1 + i, 1 \leq i \leq n;$$

$$f(u_iu_i) = \begin{cases} 2(n + 1) & if\ i = 1 \\ 2n + 2 + i\ , & if\ 2 \leq i \leq n; \end{cases}$$

$$f(u_iu_i + 1) = \begin{cases} 4n + 1 + i, & if\ 1 \leq i \leq n - 1 \\ 6n + 1, & if\ i = n. \end{cases}$$

From the definition of f, all vertex and edge labels are at most 5n + 1.The edge total means are
$m(u_iu_i+1) = i, 1 \leq i \leq n$;
$m(u_iu_i) = n + i, 1 \leq i \leq n$;
$m(u_iu_i+1) = 2n + i, 1 \leq i \leq n$;
$m(vv_i) = 3n + i, 1 \leq i \leq n$;
Hence, the edge total means form the set[1, 4n]. Therefore, f is an edge total mean labeling. An example of edge total mean labeling of closed helm graph CH6 is shown in Figure 4.

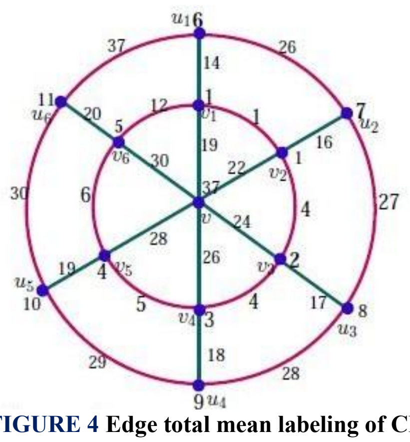

FIGURE 4 **Edge total mean labeling of CH_6**

Theorem 2.9. *The double wheel graph DW_n, $n \geq 3$ admits an edge total mean labeling.*

Proof. Define $f : V \cup E \longrightarrow [1, 6n + 1]$ as follows:

$$f(v) = 6n + 1;$$
$$f(u_i) = \begin{cases} 1, & if\ i = 1 \\ i - 1, & if 2 \leq i \leq n; \end{cases}$$

$$f(u_i u_i + 1) = \begin{cases} 1, & if\ i = 1 \\ i + 1, & if\ 2 \leq i \leq n \\ 2n, & if\ i = n \end{cases}$$

$$f(u_i) = n - 1 + i, 1 \leq i \leq n;$$
$$f(u_i u_i) = 2n + 2i, 1 \leq i \leq n;$$
$$f(u_i u_i) = \begin{cases} 1, & if\ i = 1 \\ 2i\,, & if\ 2 \leq i \leq n; \end{cases}$$
$$f(u_i u_i + 1) = \begin{cases} n + 1 + i, & if\ 1 \leq i \leq n - 1; \\ 3n + 1, & if\ i = n. \end{cases}$$

From the definition of f, all vertex and edge labels are at most 6n + 1.The edge total means are
m(vivi+1) = i, 1 ≤ i ≤ n;
m(uiui+1) = n + i, 1 ≤ i ≤ n;
m(vui) = 3n + i, 1 ≤ i ≤ n;
m(vvi) = 2n + i, 1 ≤ i ≤ n;
Hence, the edge total means form the set[1, 4n].
Therefore, *f* is an edge total mean labeling. An example of edge total mean labeling of double wheel graph DW6 is shown in Figure 5.

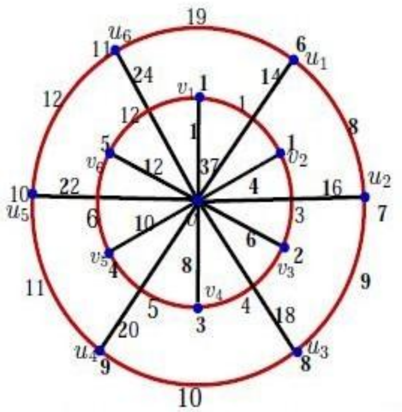

FIGURE 5 Edge total mean labeling of DW_6

Theorem 2.10. The fan graph F_n, $n \geq 2$ admits an edge total mean labeling.
Proof. Define f : V ∪ E −→ [1, 3n] as follows:

$$f(u) = 3n - 2;$$
$$f(u_i) = \begin{cases} 1, & if\ i = 1 \\ i - 1, & if\ 2 \leq i \leq n; \end{cases}$$
$$f(u_i u_i + 1) = \begin{cases} 1, & if\ i = 1; \\ i + 1, & if\ 2 \leq i \leq n - 1. \end{cases}$$
$$f(uu_i) = \begin{cases} 1, & if\ i = 1 \\ 2i, & if\ 2 \leq i \leq n. \end{cases}$$

From the definition of *f*, all vertex and edge labels are at most 3n. The edge total means are

$$m(v_i v_i + 1) = i,\ 1 \leq i \leq n;$$
$$m(uv_i) = \begin{cases} n, & if\ i = 1 \\ n + i - 1, & if\ 2 \leq i \leq n. \end{cases}$$

Hence, the edge total means form the set [1, 3n]. Therefore, f is an edge total mean labeling. An example of edge total mean labeling of fan graph F7 is shown in Figure 6.

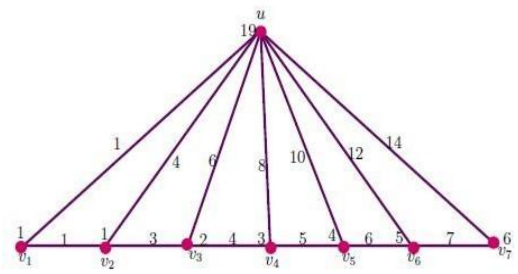

FIGURE 6 Edge total mean labeling of F_7

Theorem 2.11. *The double fan graph DF_n, $n \geq 2$ admits an edge total mean labeling.*

Proof. Define $f : V \cup E \longrightarrow [1, 4n + 1]$ as follows:

$$f(u) = 3n - 2;$$

$$f(v_1) = f(v_1v_2) = 1;$$

$$f(v_i) = \begin{cases} 1, & if\ i = 1 \\ i - 1, & if\ 2 \leq i \leq n; \end{cases}$$

$$f(u_iu_i + 1) = \begin{cases} 1, & if\ i = 1; \\ i + 1, & if\ 2 \leq i \leq n - 1; \end{cases}$$

$$f(u') = 4n - 3;$$

$$f(uu_i) = \begin{cases} 1, & if\ i = 1 \\ 2n, & if\ 2 \leq i \leq n; \end{cases}$$

$$f(v_iu') = \begin{cases} 2(n + 1), & if\ i = 1 \\ 2(n + i) + 1, & if 2 \leq i \leq n. \end{cases}$$

From the definition of f, all vertex and edge labels are at most 3n. The edge total means are
$m(v_iv_i+1) = i$, $1 \leq i \leq n$;
$m(uv_i) = n + i - 1$, $1 \leq i \leq n$;
$m(v_iu') = 2n + i - 1$, $1 \leq i \leq n$;
Hence, the edge total means form the set$[1, 3n - 1]$. Therefore, f is an edge total mean labeling. An example of edge total mean labeling of double fan graph DF8 is shown in Figure 7.

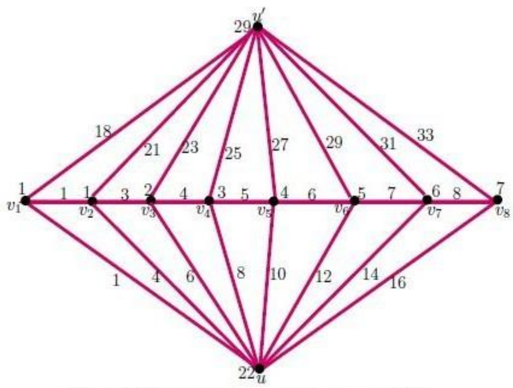

FIGURE 7 Edge total mean labeling DF_8

Theorem 2.12. The gear graph Gn, $n \geq 3$ admits an edge total mean labeling.

Proof. Define $f : V \cup E \longrightarrow [1, 5n + 1]$ as follows:

$$f(u_i) = i;\ 1 \leq i \leq n;$$

$$f(v_i) = i;\ 1 \leq i \leq n;$$

$$f(uu_i) = \begin{cases} 1, & if\ i = 1 \\ 4i - 3, & if\ 2 \leq i \leq n; \end{cases}$$

$$f(u_iu_i + 1) = \begin{cases} 4i - 1, & if\ 1 \leq i \leq n - 1 \\ 5n - 1, & if\ i = n; \end{cases}$$

$$f(uu_i) = n + 2i - 1, 1 \leq i \leq n;$$

$$f(v) = 5n + 1.$$

From the definition of f, all vertex and edge labels are at most 3n. The edge total means are
$m(v_iu_i) = 2i - 1,\ 1 \leq i \leq n$;
$m(u_iv_i+1) = 2i,\ 1 \leq i \leq n$;
$m(vv_i) = 2n + i,\ 1 \leq i \leq n$.
Hence, the edge total means form the set[1, 3n]. Therefore, f is an edge total mean labeling. An example of edge total mean labeling of gear graph G8 is shown in Figure 8.

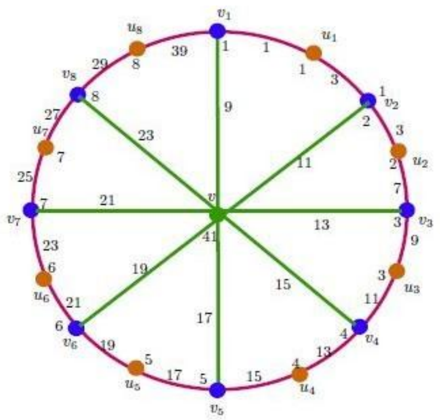

FIGURE 8 **Edge total mean labeling G_8**

Theorem 2.13. The sun graph Sn, $n \geq 3$ admits an edge total mean labeling.
Proof. Define $f : V \cup E \rightarrow [1, 4n]$ as follows:

$$f(v_i) = \begin{cases} 1, & if\ i = 1 \\ i - 1, & if\ 2 \leq i \leq n; \end{cases}$$

$$f(uu_i + 1) = \begin{cases} 1, & if\ i = 1 \\ i + 1, & 2 \leq i \leq n - 1; \\ 2n, & if\ i = n; \end{cases}$$

$$f(u_i) = \begin{cases} 2n, & if\ i = 1 \\ 2n + 1, & if\ 2 \leq i \leq n; \end{cases}$$

$$f(v_iu_i) = n + i + 1; 1 \leq i \leq n.$$

From the definition of *f*, all vertex and edge labels are at most 4n. The edge total means are
$m(vivi+1) = i,\ 1 \leq i \leq n$;
$m(viui) = n + i,\ 1 \leq i \leq n$;
Hence, the edge total means form the set[1, 2n]. Therefore, f is an edge total mean labeling. An example of edge total mean labeling of sun graph S8 is shown in Figure 9.

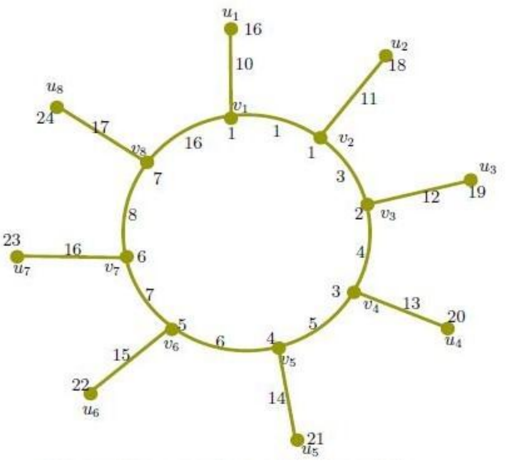

FIGURE 9 **Edge total mean labeling of S_8**

Theorem 2.14. The flower graph Fln, n ≥ 3 admits an edge total mean labeling.

Proof. Define f : V ∪ E −→ [1, 6n + 1] as follows:

$$f(v) = 5n + 1;$$
$$f(v_1) = f(v_1v_2) = 1;$$
$$f(v_i) = i - 1;\ 2 \leq i \leq n;$$
$$f(v_iv_i{+}1) = i + 1;\ 2 \leq i \leq n - 1;$$
$$f(v_iv_i{+}1) = i + 1;\ 2 \leq i \leq n - 1;$$
$$f(v_nv_1) = 2n;$$
$$f(u_i) = n - 1 + i;\ 1 \leq i \leq n;$$
$$f(v_1u_1) = 2(n + 1);$$
$$f(vv_1) = n + 1;$$
$$f(vv_i) = n + 2i,\ 2 \leq i \leq n;$$
$$f(vu_i) = 3n + 2i;\ 1 \leq i \leq n.$$

From the definition of f, all vertex and edge labels are at most 6n + 1. The edge total means are

$m(v_1v_2) = 1;$

$m(v_iv_i{+}1) = i,\ 2 \leq i \leq n - 1;$

$m(v_nv_1) = n,\ 1 \leq i \leq n;$

$m(v_1u_1) = n + 1;$

$m(v_iu_i) = n + i,\ 2 \leq i \leq n;$

$m(vv_1) = 2n + 1;$

$m(vv_i) = 2n + i,\ 2 \leq i \leq n;$

$m(vu_i) = 3n + i,\ 1 \leq i \leq n.$

Hence, the edge total means form the set [1, 4n]. Therefore, f is an edge total mean labeling. An example of edge total mean labeling of flower graph F*l8* is shown in Figure 10.

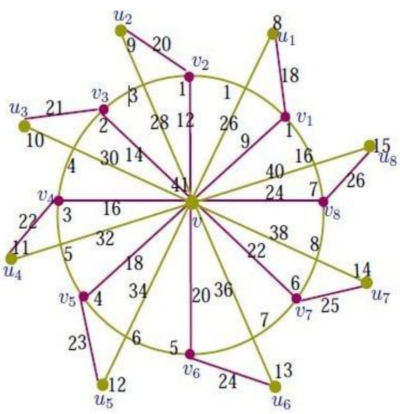

FIGURE 10 **Edge total mean labeling of *Fl_8***

REFERENCES

[1] Joseph A. Gallian, "A Dynamic Survey of Graph Labeling," The Electronic Journal of Combinatorics, pp. 1-623, 2022.

[2] Frank Harary, Graph Theory, Addison-Wesley Publishing Company, Reading, Massachusetts; London, pp. 1-284, 1972.

[3] P. Jeyanthi and A. Sudha, "Total Edge Irregularity Strength of Wheel Related Graphs of Graph Labeling," vol. 2, no. 1, pp. 45-57, 2015.

[4] P. Jeyanthi and A. Sudha, "On the total irregularity strength of some graphs," Bulletin of the International Mathematical Virtual Institute, vol. 9, no. 2, pp. 393-401, 2019.

[5] P. Jeyanthi and A. Sudha, "Total Edge Irregularity Strength of Disjoint Union of Double Wheel Graphs," Proyecciones Journal of Mathematics, vol. 35, no. 3, pp. 251-262, 2016.

[6] P.Jeyanthi and A. Sudha, "Total vertex irregularity strength of corona product of some graphs," Journal of Algorithms and Computation, vol. 48, pp. 127-140, 2016.

[7] P. Jeyanthi and A. Sudha, "Total edge irregularity strength of some families of graphs," Utilitas Mathematica, vol. 109, pp. 139-153, 2018.

[8] P. Jeyanthi and A. Sudha, " Some results on edge irregular total labeling," Bulletin of the International Mathematical Virtual Institute, vol. 9, pp. 73-91, 2019.

[9] K. Karupasamy and S. Kaleeswari, "Total Mean labelings Graphs, International Journal of Recent Technology and Engineering (IJRTE),vol. 8, no. 4S4, pp. 90-92, 2019.

[10] S. Somasundaram and R. Ponraj, " Mean labelings of Graphs," National Academy Science Letters, vol. 26, no. 7, pp. 210-213, 2003.