International Journal of Emerging Trends in Engineering and Technology
ISSN: 3107-8699 | Volume 2 Issue 2, 1-7, April-June 2026
DOI: https://doi.org/10.64137/31078699/IJETET-V2I2P101

Received: 15/03/2026 Revised: 29/03/2026 Accepted: 02/04/2026 Published: 05/04/2026

Scholastic
Research Publication

Original Article

Constrained Fuzzy Arithmetic Approach to Solve the Fully Fuzzy Transshipment Problem

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ABSTRACT: *Transportation is the distribution of goods and resources from one place to another. Most of the solving techniques for solving fully fuzzy mathematical programs are based on the standard fuzzy arithmetic operations. The constrained fuzzy arithmetic concept generates efficient solutions for many real-life applications. The transhipment problem is an extension of the transportation problem, shipping from a source to another source, shipping from a destination to another destination and shipping from a destination to any source may be allowed. This paper proposes a fuzzy constrained arithmetic approach to minimize the cost of fuzzy transportation. This paper presents an efficient algorithm for solving the transhipment problem using a fuzzy constrained arithmetic approach. At first, convert the transportation problem into an equivalent transhipment problem and then solve it by using the constrained fuzzy arithmetic algorithm. This novel method gives the minimum cost for the transhipment problem. In this paper, the solution procedure for the fully fuzzy transhipment problem using the CFA approach is explained with the help of a numerical example as another application of this algorithm.*

KEYWORDS: *Fully Fuzzy Transshipment Problem, Constrained Fuzzy Arithmetic, Fuzzy Optimization, Fuzzy Transportation Model, Fuzzy Linear Programming, Trapezoidal Fuzzy Numbers, Triangular Fuzzy Numbers, Operations Research, Supply Chain Optimization, Network Flow Problem, Uncertainty Modeling, Fuzzy Decision Making, Optimization Techniques, Computational Intelligence, Soft Computing.*

1. TRANSHIPMENT PROBLEM

In a transportation problem, shipment of a commodity takes place among sources and destinations, but instead of direct shipments to destinations, the commodity can be transported to a particular destination through one or more intermediate or transhipment points.

Each of these points in turn contributes to the other points. Thus, when the shipments pass from destination to destination and from source to source, we have a transhipment problem.

2. FULLY FUZZY TRANSSHIPMENT PROBLEM

Let we have m origins and n destinations. It is known that in a transhipment problem, any origin or destination can ship to any other origin or destination. For convenience, the origins can be numbered successively. (i.e.) from m+1 to m+n.

Let a_i be the quantities available at the origins and b_j be the demands at the destinations. It is known that the necessary and sufficient condition for the existence of a feasible solution is,

$$\sum_{i=1}^{S} a_i = \sum_{j=1}^{D} b_j$$

3. MATHEMATICAL FORMULATION OF FULLY FUZZY TRANSHIPMENT PROBLEM (FFTSP):

The mathematical formulation of a fully fuzzy transhipment problem with triangular fuzzy numbers can be expressed as follows:

Minimize $$\tilde{z} \approx \sum_{i=1}^{S+D} \sum_{j=1, j\neq i}^{S+D} \tilde{c}_{ij} \otimes \tilde{x}_{ij} \qquad (1)$$

Subject to;

$$\sum_{j=1}^{S+D} x_{ij} = Cap_i + T, i = 1,2...S \qquad (2)$$

$$\sum_{j=1}^{S+D} x_{ij} = T, \qquad i = S+1, S+2 \ldots\ldots S+D \tag{3}$$

$$\sum_{i=1}^{S+D} x_{ij} = T, \qquad j = 1, 2 \ldots\ldots S \tag{4}$$

$$\sum_{i=1}^{S+D} x_{ij} = De_j + T, \; j = S+1, S+2 \ldots\ldots S+D \tag{5}$$

Where $x_{ij} \geq 0, i, j = 1, 2 \ldots\ldots\ldots S+D$ (6)

The algorithmic step of the CFA based method to solve FFTSPs for the risk averse decision makers is given as follows:
Step 1: The conjugate model of the given FFTP can be written as follows without using any fuzzy ranking functions or auxiliary variables

Minimize $\displaystyle\sum_{i=1}^{S+D} \sum_{j=1, j\neq i}^{S+D} \tilde{c}_{ij} \otimes \tilde{x}_{ij}$ (7)

$$\left(Cap_i \ominus \sum_{j=1}^{S+D} x_{ij} \right)_{\sum_j x_{ij} \leq Cap_i} \geq 0 \qquad \forall \text{ i} \tag{8}$$

$$\left(Cap_i \ominus \sum_{j=1}^{S+D} x_{ij} + T \right)_{\sum_j x_{ij} \leq Cap_i} \geq 0 \qquad \forall \text{ i} \tag{9}$$

$$\left(De_j \ominus \sum_{i=1}^{S+D} x_{ij} \right)_{\sum_i x_{ij} \geq De_j} \leq 0 \qquad \forall \text{j} \tag{10}$$

$$\left(De_j \ominus \sum_{i=1}^{S+D} x_{ij} + T \right)_{\sum_i x_{ij} \geq De_j} \leq 0 \qquad \forall \text{j} \tag{11}$$

Step 2: the fuzzy arithmetic operations with requisite fuzzy equality constraints in conjugate model can be performed as follows. Actually, these operations are performed among the fuzzy capacities, demands and transshipment amount as given by Eqs. (12) and (13).

$$\left(Cap_i * \sum_{j=1}^{S+D} x_{ij} \right)_E^{\alpha} = \left\{ Cap_i * \sum_j x_{ij} \,\middle|\, \left\langle Cap_i, \sum_j x_{ij} \right\rangle \in \left(Cap_i^{\alpha} \times \sum_{j=1}^{S+D} x_{ij}^{\alpha} \right) \bigcap E^{\alpha} \right\} \tag{12}$$

$$\left(De_j * \sum_{i=1}^{S+D} x_{ij} \right)_E^{\alpha} = \left\{ De_j * \sum_i x_{ij} \,\middle|\, \left\langle De_j, \sum_i x_{ij} \right\rangle \in \left(De_j^{\alpha} \times \sum_{i=1}^{S+D} x_{ij}^{\alpha} \right) \bigcap E^{\alpha} \right\} \tag{13}$$

Step 3: If all of the uncertain parameters and variables for the given FFTSP are symmetric or non-symmetric triangular fuzzy intervals, then we can obtain the following results of the fuzzy subtraction operations under fuzzy equality constraints for remaining capacities and demands:

$$\left[Cap_i \ominus \sum_{j=1}^{S+D} x_{ij} + T \right]_E^{\alpha} = \tag{14}$$

$$\underline{Cap_i}(\alpha) - \underline{\sum_{j=1}^{S+D} x_{ij}}(\alpha) - \Delta\left(\frac{a_i + b_i}{2} \right)\sqrt{2}(1-\alpha) + T(\alpha), \overline{Cap_i}(\alpha) - \overline{\sum_{j=1}^{S+D} x_{ij}}(\alpha) + \Delta\left(\frac{a_i + b_i}{2} \right)\sqrt{2}(1-\alpha) + T(\alpha)$$

$$\left[De_j \ominus \sum_{i=1}^{S+D} x_{ij} + T\right]_E^{\alpha} =$$

$$\left[\underline{De}_j(\alpha) - \underline{\sum_{i=1}^{S+D} x_{ij}(\alpha)} - \Delta\left(\frac{c_j + d_j}{2}\right)\sqrt{2}(1-\alpha) + T(\alpha), \overline{De}_j(\alpha) - \overline{\sum_{i=1}^{S+D}} x_{ij}(\alpha) + \Delta\left(\frac{c_j + d_j}{2}\right)\sqrt{2}(1-\alpha) + T(\alpha)\right]^{\alpha} \quad (15)$$

Step 4: The crisp equivalent form of the conjugate model can be written as follows by using the results of arithmetic operations obtained from step 3. As mentioned previously, the crisp Lp problem of the stated FFP with fuzzy intervals can be formulated as in Eqns. (16)-(25).

Minimize

$$\left[\underline{C}_{ij}(\alpha) \otimes \underline{x}_{ij}(\alpha) + \overline{C}_{ij}(\alpha) \otimes \overline{x}_{ij}(\alpha)\right] \quad (16)$$

Subject to;

$$\underline{Cap}_i(\alpha) - \overline{\sum_{j=1}^{S+D}} x_{ij}(\alpha) \geq -\Delta\sqrt{2}\left(\frac{\overline{Cap}_i(\alpha) - \underline{Cap}_i(\alpha)}{2}\right) \quad \forall i \quad (17)$$

$$\underline{Cap}_i(\alpha) - \overline{\sum_{j=1}^{S+D}} x_{ij}(\alpha) + T(\alpha) \geq -\Delta\sqrt{2}\left(\frac{\overline{Cap}_i(\alpha) - \underline{Cap}_i(\alpha)}{2}\right) \quad \forall i \quad (18)$$

$$\overline{Cap}_i(\alpha) - \underline{\sum_{j=1}^{S+D}} x_{ij}(\alpha) \geq \Delta\sqrt{2}\left(\frac{\overline{Cap}_i(\alpha) - \underline{Cap}_i(\alpha)}{2}\right) \quad \forall i \quad (19)$$

$$\overline{Cap}_i(\alpha) - \underline{\sum_{j=1}^{S+D}} x_{ij}(\alpha) + T(\alpha) \geq \Delta\sqrt{2}\left(\frac{\overline{Cap}_i(\alpha) - \underline{Cap}_i(\alpha)}{2}\right) \quad \forall i \quad (20)$$

$$\underline{De}_j(\alpha) - \overline{\sum_{i=1}^{S+D}} x_{ij}(\alpha) \leq -\Delta\sqrt{2}\left(\frac{\overline{De}_j(\alpha) - \underline{De}_j(\alpha)}{2}\right) \quad \forall j \quad (21)$$

$$\underline{De}_j(\alpha) - \overline{\sum_{i=1}^{S+D}} x_{ij}(\alpha) + T(\alpha) \leq -\Delta\sqrt{2}\left(\frac{\overline{De}_j(\alpha) - \underline{De}_j(\alpha)}{2}\right) \quad \forall j \quad (22)$$

$$\overline{De}_j(\alpha) - \underline{\sum_{i=1}^{S+D}} x_{ij}(\alpha) \leq \Delta\sqrt{2}\left(\frac{\overline{De}_j(\alpha) - \underline{De}_j(\alpha)}{2}\right) \quad \forall j \quad (23)$$

$$\overline{De}_j(\alpha) - \underline{\sum_{i=1}^{S+D}} x_{ij}(\alpha) + T(\alpha) \leq \Delta\sqrt{2}\left(\frac{\overline{De}_j(\alpha) - \underline{De}_j(\alpha)}{2}\right) \quad \forall j \quad (24)$$

$$\underline{x}_{ij}(\alpha) \geq and\, \overline{x}_{ij}(\alpha) \geq 0 \quad \forall i, \forall j \quad (25)$$

These constraints involved by the above step can also be presented in a more compact manner by using the widths of the triangular fuzzy numbers, a_i, b and c_j, d_j as follows:

$$\underline{Cap}_i(\alpha) - \overline{\sum_{j=1}^{S+D}} x_{ij}(\alpha) + T(\alpha) \geq -\Delta\left(\frac{a_i + b_j}{2}\right)\sqrt{2}(1-\alpha) \quad \forall i \quad (26)$$

$$\overline{Cap}_i(\alpha) - \underline{\sum_{j=1}^{S+D}} x_{ij}(\alpha) + T(\alpha) \geq \Delta\left(\frac{a_i + b_j}{2}\right)\sqrt{2}(1-\alpha) \quad \forall i \quad (27)$$

$$\underline{De}_j(\alpha)-\overline{\sum_{i=1}^{S+D}x_{ij}(\alpha)}+T(\alpha)\leq-\Delta\left(\frac{c_j+d_j}{2}\right)\sqrt{2}(1-\alpha)\qquad\forall j \tag{28}$$

$$\overline{De}_j(\alpha)-\underline{\sum_{i=1}^{S+D}x_{ij}(\alpha)}+T(\alpha)\leq\Delta\left(\frac{c_j+d_j}{2}\right)\sqrt{2}(1-\alpha)\qquad\forall j \tag{29}$$

$$\underline{x_{ij}}(\alpha)\geq and\,\overline{x_{ij}}(\alpha)\geq 0\qquad\forall i,\forall j \tag{30}$$

Step 5: By substituting the Eqs. (9), (10), (12) and (13) into the above end points, the parametric final crisp LP problem can be obtained as in Eqs. (31) – (36).

$$\text{MinZ=}\quad\left[\sum_{i=1}^{S+D}\sum_{j=1}^{S+D}\left(C_{ij}^{p}+\left(C_{ij}^{m}-C_{ij}^{p}\right)\alpha\right)\left(x_{ij}^{p}+\left(x_{ij}^{m}-x_{ij}^{p}\right)\alpha\right)\sum_{i=1}^{S+D}\sum_{j=1}^{S+D}\left(C_{ij}^{o}-\left(C_{ij}^{o}-C_{ij}^{m}\right)\alpha\right)\left(x_{ij}^{o}-\left(x_{ij}^{o}-x_{ij}^{m}\right)\alpha\right)\right] \tag{31}$$

Subject to;

$$Cap_i^p+\left(Cap_i^m-Cap_i^p\right)\alpha-\left(\sum_{j=1}^{S+D}x_{ij}^{o}-\left(\sum_{j=1}^{S+D}x_{ij}^{m}-\sum_{j=1}^{S+D}x_{ij}^{m}\right)\alpha\right)+T(\alpha)\geq-\Delta\left(\frac{a_i+b_i}{2}\right)\sqrt{2}\left(1-\alpha\right)\ \forall i \tag{32}$$

$$Cap_i^o-\left(Cap_i^o-Cap_i^m\right)\alpha-\left(\sum_{j=1}^{S+D}x_{ij}^{p}+\left(\sum_{j=1}^{S+D}x_{ij}^{m}-\sum_{j=1}^{S+D}x_{ij}^{p}\right)\alpha\right)+T(\alpha)\geq\Delta\left(\frac{a_i+b_i}{2}\right)\sqrt{2}\left(1-\alpha\right)\quad\forall i \tag{33}$$

$$De_j^p+\left(De_j^m-De_j^p\right)\alpha-\left(\sum_{i=1}^{S+D}x_{ij}^{o}-\left(\sum_{i=1}^{S+D}x_{ij}^{o}-\sum_{i=1}^{S+D}x_{ij}^{m}\right)\alpha\right)+T(\alpha)\geq-\Delta\left(\frac{c_j+d_j}{2}\right)\sqrt{2}\left(1-\alpha\right)\quad\forall j \tag{34}$$

$$De_j^o-\left(De_j^o-De_j^m\right)\alpha-\left(\sum_{i=1}^{S+D}x_{ij}^{p}+\left(\sum_{i=1}^{S+D}x_{ij}^{m}-\sum_{i=1}^{S+D}x_{ij}^{p}\right)\alpha\right)+T(\alpha)\geq\Delta\left(\frac{c_j+d_j}{2}\right)\sqrt{2}\left(1-\alpha\right)\ \forall j \tag{35}$$

$$x_{ij}^{o}\geq x_{ij}^{m},\quad x_{ij}^{m}\geq x_{ij}^{p},x_{ij}^{p}\geq 0\qquad\forall i\ ,\forall j \tag{36}$$

4. NUMERICAL EXAMPLE

Type: 1

The given problem is the numerical example for type-1,which is balanced FFTSP problemUsing LINGO 13.0 software , the optimal solution for the type-I problem is obtained as follows:

$\tilde{x}_{12}^{*}=(0,0,0)$, $\tilde{x}_{13}^{*}=(0\ ,0,\ 0)$, $\tilde{x}_{14}^{*}=(6.2,7,7)$, $\tilde{x}_{15}^{*}=(0,\ 0,\ 0)$,

$\tilde{x}_{24}^{*}=(0,\ 0,\ 0)$, $\tilde{x}_{25}^{*}=(0,\ 0,\ 0.5)$, $\tilde{x}_{26}^{*}=(4.2,\ 5,\ 5.9)$, $\tilde{x}_{27}^{*}=(7.8,\ 9,\ 9)$,

$\tilde{x}_{31}^{*}=(0,\ 0,\ 0)$, $\tilde{x}_{32}^{*}=(0,0,0)$, $\tilde{x}_{34}^{*}=(0,0,0.8)$, $\tilde{x}_{35}^{*}=(8.9,10,10.6)$

$\tilde{x}_{36}^{*}=(1.3,2,2)$, $\tilde{x}_{37}^{*}=(0,0,0.4)$, $\tilde{x}_{41}^{*}=(0,0,0)$, $\tilde{x}_{42}^{*}=(0,0,0)$,

$\tilde{x}_{43}^{*}=(0,0,0)$, $\tilde{x}_{45}^{*}=(0,0,0)$, $\tilde{x}_{46}^{*}=(0,0,0)$, $\tilde{x}_{47}^{*}=(0,0,0)$,

$\tilde{x}_{51}^{*}=(0,0,0)$, $\tilde{x}_{52}^{*}=(0,0,0)$, $\tilde{x}_{53}^{*}=(0,0,0)$, $\tilde{x}_{54}^{*}=(0,0,0)$,

$\tilde{x}_{56}^{*}=(0,0,0)$, $\tilde{x}_{57}^{*}=(0,0,0)$, $\tilde{x}_{61}^{*}=(0,0,0)$, $\tilde{x}_{62}^{*}=(0,0,0)$,

$\tilde{x}_{63}^{*}=(0,0,0)$, $\tilde{x}_{64}^{*}=(0,0,0)$, $\tilde{x}_{65}^{*}=(0,0,0)$, $\tilde{x}_{67}^{*}=(0,0,0)$,

$\tilde{x}_{71}^{*}=(0,0,0)$, $\tilde{x}_{72}^{*}=(0,0,0)$, $\tilde{x}_{73}^{*}=(0,0,0)$, $\tilde{x}_{74}^{*}=(0,0,0)$,

$\tilde{x}_{75}^{*}=(0,0,0)$, $\tilde{x}_{76}^{*}=(0,0,0)$

TABLE 1 **Triangular Fuzzy Cost Matrix for Supply and Demand Nodes**

	S_1	S_2	S_3	D_1	D_2	D_3	D_4	Supply
S_1	-	(9.2,10,11)	(18,19.2,20)	(8,10,10.8)	(20.4,22,24)	(8,10,10.6)	(18.8,20,22)	(7.2,8,8.8)
S_2	(9.2,10,11)	-	(14,15.2,16)	(14,15,16)	(18.2,20,22)	(10,12,13)	(6,8,8.8)	(12,14,16)
S_3	(18,19.2,20)	(14,15.2,16)	-	(18.4,20,21)	(9.6,12,13)	(7.8,10,10.8)	(14,15,16)	(10.2,12,13.8)
D_1	(8,10,10.8)	(8,10,10.8)	(8,10,10.8)	-	(14,15,16)	(8,10,10.2)	(9,10.2,11)	-
D_2	(20.4,22,24)	(20.4,22,24)	(20.4,22,24)	(14,15,16)	-	(9.6,10,10.2)	(9,10,11.2)	-
D_3	(8,10,10.6)	(8,10,10.6)	(8,10,10.6)	(8,10,10.2)	(9.6,10,10.2)	-	(18,19,20)	-
D_4	(18.8,20,22)	(18.8,20,22)	(18.8,20,22)	(9,10.2,11	(9,10,11.2)	(18,19,20)	-	-
D	-	-	-	(6.2,7,7.8)	(8.9,10,11.1)	(6.5,8,9.5)	(7.8,9,10.2)	

The optimal value of the problem is achieved by putting $\tilde{x}^*$ in $\tilde{c}\tilde{x}$ as follows:

$$(\tilde{c}\,\tilde{x}^*) = \left((cx^*)^p,\ (cx^*)^m,\ (cx^*)^o \right)$$

$$= \left(\sum_{i=1}^{S}\sum_{j=1}^{D}(c_{ij}x_{ij}^*)^p\,,\ \sum_{i=1}^{S}\sum_{j=1}^{D}(c_{ij}x_{ij}^*)^m\,,\ \sum_{i=1}^{S}\sum_{j=1}^{D}(c_{ij}x_{ij}^*)^o \right)$$

$$= \mathbf{(235.98,\ 352,\ 458.21)}$$

Type:2

The given problem is the numerical example for type-2, which is balanced FFTSP problem Using LINGO 13.0 software, the optimal solution for the type-II problem is obtained as follows:

$\tilde{x}_{12}^* = (0,0,0)$, $\tilde{x}_{13}^*$ =(0, 0, 0), $\tilde{x}_{14}^*$ =(300,300,300), $\tilde{x}_{15}^*$ = (0, 0, 0),
$\tilde{x}_{16}^* = (300,300,300)$, $\tilde{x}_{17}^* = (0,0,0)$, $\tilde{x}_{21}^* = (0,0,0)$, $\tilde{x}_{23}^* = (0,0,0)$,
$\tilde{x}_{24}^*$= (0, 0, 0) , $\tilde{x}_{25}^*$ = (0, 0, 0.5), $\tilde{x}_{26}^*$= (0,0,0), $\tilde{x}_{27}^*$= (350,350,350),
$\tilde{x}_{31}^*$= (0, 0, 0), $\tilde{x}_{32}^*$=(0,0,0), $\tilde{x}_{34}^*$=(0,0,0), $\tilde{x}_{35}^*$=(0,0,0)
$\tilde{x}_{36}^* = (0,0,0)$, $\tilde{x}_{37}^* = (0,0,0)$, $\tilde{x}_{41}^* = (0,0,0)$, $\tilde{x}_{42}^* = (0,0,0)$,
$\tilde{x}_{43}^* = (0,0,0)$, $\tilde{x}_{45}^* = (0,0,0)$, $\tilde{x}_{46}^* = (0,0,0)$, $\tilde{x}_{47}^* = (0,0,0)$,
$\tilde{x}_{51}^* = (0,0,0)$, $\tilde{x}_{52}^* = (0,0,0)$, $\tilde{x}_{53}^* = (0,0,0)$, $\tilde{x}_{54}^* = (0,0,0)$,
$\tilde{x}_{56}^* = (0,0,0)$, $\tilde{x}_{57}^* = (0,0,0)$, $\tilde{x}_{61}^* = (0,0,0)$, $\tilde{x}_{62}^* = (0,0,0)$,
$\tilde{x}_{63}^* = (0,0,0)$, $\tilde{x}_{64}^* = (0,0,0)$, $\tilde{x}_{65}^* = (0,0,0)$, $\tilde{x}_{67}^* = (0,0,0)$,
$\tilde{x}_{71}^* = (0,0,0)$, $\tilde{x}_{72}^* = (0,0,0)$, $\tilde{x}_{73}^* = (0,0,0)$, $\tilde{x}_{74}^* = (0,0,0)$,
$\tilde{x}_{75}^* = (0,0,0)$, $\tilde{x}_{76}^* = (0,0,0)$

TABLE 2 **Fuzzy Supply–Demand Transportation Matrix Using Triangular Fuzzy Numbers**

	S_1	S_2	S_3	D_1	D_2	D_3	D_4	Supply
S_1	-	(18,20,22)	(22,25,27)	(17,19,21)	(15,16,17)	(15,16,17)	(16,18,20)	(550,650,800)
S_2	(18,20,22)	-	(14,15,16)	(17,19,21)	(12,14,16)	(14,16,18)	(11,13,15)	(550,700,850)
S_3	(22,25,27)	(14,15,16)	-	(22,25,27)	(18,20,22)	(20,21,23)	(20,22,24)	(600,800,1000)
D_1	(17,19,21)	(17,19,21)	(22,25,27)	-	(14,15,16)	(8,10,10.2)	(9,10.2,11)	-
D_2	(15,16,17)	(12,14,16)	(18,20,22)	(14,15,16)	-	(9.6,10,10)	(9,10,11.2)	-
D_3	(15,16,17)	(14,16,18)	(20,21,23)	(8,10,10.2)	(9.6,10,10)	-	(18,19,20)	-
D_4	(16,18,20)	(11,13,15)	(20,21,23)	(9,10.2,11	(9,10,11.2)	(18,19,20)	-	-
D	-	-	-	(250,300,350)	(300,350,400)	(250,300,350)	(300,350,400)	

The optimal value of the problem is achieved by putting $\tilde{x}^*$ in $\tilde{c}\tilde{x}$ as follows:

$$(\tilde{c}\ \tilde{x}^*) = (\ (cx^*)^p,\ (cx^*)^m,\ (cx^*)^o)$$

$$= \left(\sum_{i=1}^{S}\sum_{j=1}^{D}(c_{ij}x_{ij}^*)^p\ ,\ \sum_{i=1}^{S}\sum_{j=1}^{D}(c_{ij}x_{ij}^*)^m\ ,\ \sum_{i=1}^{S}\sum_{j=1}^{D}(c_{ij}x_{ij}^*)^o \right)$$

= (16950, 17900, 21790

5. CONCLUSION

In this paper, the methodology to solve the fuzzy transportation problem using CFA is discussed, where all of the parameters and decision variables are stated as triangular fuzzy numbers. Also, a method based on the CFA concept is proposed for solving fuzzy transhipment problems. This method provided fuzzy acceptable solutions for “risk seekers” with a high degree of uncertainty. Moreover, Constrained Fuzzy Arithmetic (CFA) can be considered more useful than standard fuzzy arithmetic in terms of providing applicable solutions for many practical applications.

Furthermore, the given method assists decision makers with yielding fuzzy efficient solutions under different uncertainty levels (α-cuts) and crisp solutions are a challenging task and may not be possible due to several uncertainties as discussed throughout this project. Therefore, it may always be desirable to provide fuzzy solutions by taking into consideration the decision maker’s risk attitude in such occasions. More reliable, applicable and information-efficient solutions can be presented to the decision makers under varying demand and capacity conditions using this method. Also, the optimal cost obtained by the fuzzy transhipment problem is less than that of the fuzzy transportation problem. Therefore, transhipment problems ensure cost-effective movement of the products from one place to another for the risk-averse decision maker for type-III, in which the risk-free solutions are available in different α-levels.

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